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Chairux Algorithm for Divisibility Test

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This paper presents a novel Chairux Algorithm for the divisibility test. The test is based on an arbitrary integer via the concepts of Bezout's identity and the Euclidean algorithm. Examples demonstrating the effectiveness of the proposed algorithm indicate its simplicity, efficiency, and flexibility compared to the existing state-of-the-art algorithms.

Keywords: Chairux algorithm; bezout identity; euclidean algorithm; period length.

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16.

1 INTRODUCTION

The problem of testing the divisibility of one integer by another has been of interest to mathematicians for centuries [1, 2, 3, 4, 5, 6]. Traditional methods, such as long division, require complex calculations and repeated subtractions, which can be time-consuming and error-prone. This paper presents a new method for testing divisibility called the Chairux Algorithm. The algorithm is based on the well-known concepts of Bezout's identity and the Euclidean algorithm and provides a simple and efficient approach to testing divisibility. Before presenting Chairux Algorithm, a review of the two fundamental concepts on which it is

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based: Bezout's identity and the Euclidean algorithm is presented.

1.1 Bezout's Identity

Bezout's identity is a fundamental theorem stated as follows: For any two integers a and b, not both zero, there exist integers x and y such that

$$ax + by = \gcd(a, b) \tag{1.1}$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b. This identity is useful in many areas of mathematics, including cryptography and algebraic geometry. For instance, it provides a powerful tool for solving linear Diophantine equations of the form $ax + by = c$, where a, b, and c are integers [7]. By applying the extended Euclidean algorithm to find x and y that satisfy Bézout's identity, we can determine whether a solution exists and find all possible solutions [8]. It gives a constructive method for finding the greatest common divisor (GCD) of two integers a and b (as is seen in the proposed algorithm in the paper). The coefficients x and y obtained through Bézout's identity satisfy $ax + by = \gcd(a, b)$ and express the GCD as a linear combination of a and b. It is useful in finding modular inverses. If a and m are coprime integers, Bézout's identity guarantees the existence of x and y such that $ax + my = 1$ [9]. By taking this equation modulo m, we obtain $ax \equiv 1 \pmod{m}$, indicating that x is the modular inverse of a modulo m. This is particularly useful in modular arithmetic and cryptographic algorithms. Bézout's identity plays a fundamental role in the Euclidean algorithm, which efficiently computes the GCD of two numbers [10]. The algorithm repeatedly applies the division algorithm and uses Bézout's identity to express the remainders as linear combinations of the original numbers. This process continues until the remainder becomes zero, yielding the GCD.

1.2 Euclidean Algorithm

The Euclidean algorithm is efficient for finding two integers' GCD [10]. It is based on the observation that the GCD of two numbers equals the GCD of the smaller number, and the remainder is obtained when dividing the more significant number by the smaller number. Given two positive integers a and b (with $a > b$), we perform the following steps:

1. Divide a by b and obtain the remainder r. The algorithm terminates if r is zero and the GCD is b. Otherwise, proceed to the next step.
2. Set a equal to b and b equal to r.
3. Repeat steps 1 and 2 until the remainder becomes zero.

Once the remainder becomes zero, the algorithm terminates, and the GCD is the value of b obtained in the previous step. The Euclidean algorithm is highly efficient and has a time complexity of $O(\log(\min(a, b)))$. This enables its use in various applications, such as simplifying fractions: The GCD of the numerator and denominator of a fraction can be found using the Euclidean algorithm. Dividing the numerator and denominator by their GCD simplifies the fraction to its lowest terms. They are also valuable for checking coprimality, where two integers are coprime if their GCD is 1. The Euclidean algorithm can efficiently determine whether two numbers are coprime. Modular arithmetic, where the Euclidean algorithm is used to find modular inverses. If a and m are coprime integers, the Euclidean algorithm can be applied to find x and y such that $ax + my = 1$. This equation modulo m yields $ax \equiv 1 \pmod{m}$, indicating that x is the modular inverse of a modulo m.

2 PROPOSED CHAIRUX ALGORITHM

2.1 Introduction

Let N be a divisor, given by $N = 10x+y$.Then $\exists$ a pair of integers (α,β) s.t $\alpha x+\beta y = N$, $\forall$ $\alpha, \beta \in \mathbb{Z}$ and $|\alpha| < 10$. The integers (α, β) are known as Chairux coefficient for (x, y).

Theorem 2.1. *Let* $N = 10x + y \in \mathbb{N}$.

if $\alpha = 10 - y$ *and* $\beta = x + 1$*, then*

$\alpha x + \beta y = N$

Proof. $\alpha x + \beta y$=$(10 - y)x + (x + 1)y$

=$10x - yx + yx + y$

=$10x + y$ □

2.2 Chairux Identity

let f_N be a continuous function of N, then a function $f_N(\alpha, \beta)$ is known as chairux identity function if:

$f_N(\alpha, \beta) = \alpha x + \beta y = N.$

chairux identity table

Recall $N = 10x + y = \alpha x + \beta y$

$10x - \alpha x = \beta y - y$

$x(10 - \alpha) = y(\beta - 1)$

$xy = yx$

$\therefore y = 10 - \alpha$ and $x = \beta - 1$

proof 1

N=10x+y

$N = 10(\beta - 1) + 10 - \alpha$

$N = 10\beta - 10 + 10 - \alpha$

$\therefore N = 10\beta - \alpha$

proof 2

$N = \alpha x + \beta y$

$N = \alpha(\beta - 1)$+$\beta(10 - \alpha)$

$N = \alpha\beta - \alpha + 10\beta - \beta\alpha$

$\therefore N = 10\beta - \alpha$

2.3 Chairux Minimal Iteration

Defination:Let N be a divisor and given by $N = 10x + y$, then chairux minimal iteration coefficient for N happen when then number of steps of testing divisibility is minimal.

Table 1. Chairux identity table

(α,β)	1	2	3	4	5	6	7	8	9	10
1	09	19	29	39	49	59	69	79	89	99
2	08	18	28	38	48	58	68	78	88	98
3	07	17	27	37	47	57	67	77	87	97
4	06	16	26	36	46	56	66	76	86	96
5	05	15	25	35	45	55	65	75	85	95
6	04	14	24	34	44	54	64	74	84	94
7	03	13	23	33	43	53	63	73	83	93
8	02	12	22	32	42	52	62	72	82	92
9	01	11	21	31	41	51	61	71	81	91
10	00	10	20	30	40	50	60	70	80	90

Chairux minimum iteration is given by:

$$(\alpha,\beta)=\begin{cases}[(10-y),(1+x)] & : y\geq 5\\ [(y^2-6y+10),(6x-xy+1)] & : y\leq 5\end{cases}$$

Proof. recall

1) $\alpha = 10 - ky$ and $\beta = 1 + kx$ for $y \geq 5$, $k = 1$

$\therefore (\alpha,\beta) = [(10-y),(1+x)]$ for $y \geq 5$

2) for $y \leq 5$, $k = (6-y)$

i $\alpha = 10-(6-y)y$ and $\beta = 1+(6-y)x$

$\alpha = 10-6y+y^2$

$\alpha = y^2-6y+10\cdots *$

ii $\beta = 1+(6-y)x$

$\beta = 1+6x-xy$

$\beta = 6x-xy+1$

$\therefore (\alpha,\beta) = [(y^2-6y+10),(6x-xy+1)]$ □

Example

find chairux minimum coefficient of the following number.

a) 14 c) 17

b) 23 d) 54

Solution

a) 14

Recall $N = 10x + y$

$14 = 10 \times 1 + 4$

$y = 4 < 5$

since $y < 5$

$\alpha = y^2 - 6y + 10 = 4^2 - 6 \times 4 + 10 =$ **2**

$\beta = 6x - xy + 1 = 6 \times 1 - 1 \times 4 + 1 =$ **3**

$\therefore (\alpha, \beta) =$ **(2,3)**

Proof $N = \alpha x + \beta y = 2 \times 1 + 3 \times 4 = 14$

b) 23

$N = 10x + y$

$23 = 10 \times 2 + 3$

$y = 3 < 5$

since $y < 5$

$\alpha = y^2 - 6y + 10 = 3^2 - 6 \times 3 + 10 =$ **1**

$\beta = 6x - xy + 1 = 6 \times 2 - 2 \times 3 + 1 =$ **7**

$\therefore (\alpha, \beta) =$ **(1,7)**

Proof $N = \alpha x + \beta y = 1 \times 2 + 7 \times 3 = 23$

c) 17

$N = 10x + y$

$17 = 10 \times 1 + 7$

$y = 7 > 5$

since $y > 5$

$\alpha = 10 - y = 10 - 7 =$ **3**

$\beta = x + 1 = 1 + 1 =$ **2**

$\therefore (\alpha, \beta) =$ **(3,2)**

Proof $N = \alpha x + \beta y = 3 \times 1 + 2 \times 7 = 17$

2.4 Properties of Chairux Identity

1) $f_N(\alpha_a, \beta_a) \pm f_N(\alpha_b, \beta_b) = f_N[(\alpha_a \pm \alpha_b), (\beta_a \pm \beta_b)]$.

2) $kf_N(\alpha, \beta) = f_N(k\alpha, k\beta)$.

3) $f_N(\alpha, -\beta) = -f_N(-\alpha, \beta) = |f_N(-\alpha, \beta)|$.

Proof. suppose $A = (\alpha_a, \beta_a)$ and $B = (\alpha_b, \beta_b)$ s.t A, B $\in N$

then

$f_N(\alpha_a, \beta_a) + f_N(\alpha_b, \beta_b) = \lambda N \; \forall \lambda \in \mathbb{Z}$.

Recall $f_N(\alpha, \beta)$ = $\alpha x + \beta y = N$

$f_N(\alpha_a, \beta_a)$ = $\alpha_a x + \beta_a y = N \cdots *$

$f_N(\alpha_b, \beta_b)$ = $\alpha_b x + \beta_b y = N \cdots **$

by adding eqn * and ** we have:

$f_N(\alpha_a, \beta_a) + f_N(\alpha_b, \beta_b)$=$(\alpha_a x + \beta_a y)$+$(\alpha_b x + \beta_b y)$

$f_N(\alpha_a, \beta_a) + f_N(\alpha_b, \beta_b)$=$(\alpha_a x + \alpha_b x)$+$(\beta_a y + \beta_b y)$

$f_N(\alpha_a, \beta_a) + f_N(\alpha_b, \beta_b)$=$(\alpha_a + \alpha_b)x$+$(\beta_a + \beta_b)y \cdots ***$

recall $f_N(\alpha, \beta) = \alpha x + \beta y$

let $(\alpha_a + \alpha_b) = \alpha_m$ and $(\beta_a + \beta_b) = \beta_m$

then eqn *** become:

$f_N(\alpha_a, \beta_a) + f_N(\alpha_b, \beta_b) = \alpha_m x + \beta_m y$

$\alpha_m x + \beta_m y = f_N(\alpha_m, \beta_m)$

$\therefore f_N(\alpha_m, \beta_m) = \alpha_m x + \beta_m y = N$

□

Different chairux coefficient may have the same divisor.

Principle of convergence depend on α, that is as $\alpha \to 0$ chairux coeffient converge faster to N.

1) **alpha**

$\alpha_0 = 2^0\alpha_0 - (2^0 - 1)10 \Longrightarrow 2^0\alpha_0 - 10 \times 2^0 + 10$

$\alpha_1 = 2^1\alpha_0 - (2^1 - 1)10 \Longrightarrow 2^1\alpha_0 - 10 \times 2^1 + 10$

$\alpha_2 = 2^2\alpha_0 - (2^2 - 1)10 \Longrightarrow 2^2\alpha_0 - 10 \times 2^2 + 10$

$\alpha_3 = 2^3\alpha_0 - (2^3 - 1)10 \Longrightarrow 2^3\alpha_0 - 10 \times 2^3 + 10$

$\alpha_n = 2^n\alpha_0 - (2^n - 1)10 \Longrightarrow 2^n\alpha_0 - 10 \times 2^n + 10$

$\therefore \alpha_n \Longrightarrow 2^n(\alpha_0 - 10) + 10$

2) **beta**

$\beta_0 = 2^0\beta_0 - (2^0 - 1) \Longrightarrow 2^0\beta_0 - 2^0 + 1$

$\beta_1 = 2^1\beta_0 - (2^1 - 1) \Longrightarrow 2^1\beta_0 - 2^1 + 1$

$\beta_2 = 2^2\beta_0 - (2^2 - 1) \Longrightarrow 2^2\beta_0 - 2^2 + 1$

$\beta_3 = 2^3\beta_0 - (2^3 - 1) \Longrightarrow 2^3\beta_0 - 2^3 + 1$

$\beta_n = 2^n\beta_0 - (2^n - 1) \Longrightarrow 2^n\beta_0 - 2^n + 1$

$\therefore \beta_n \Longrightarrow 2^n(\beta_0 - 1) + 1$

In general **(**α_n**,**β_n**)=[(**$2^n(\alpha_0 - 10) + 10$**),(**$2^n(\beta_0 - 1) + 1$**)]** $\forall n \in \mathbb{Z}$

Table 2. Chairux coefficient with same identity

(α_0, β_0)	(α_1, β_1)
(6,2)	(2,3)
(6,3)	(2,5)
(6,4)	(2,7)
(6,5)	(2,9)
(7,2)	(4,3)
(7,3)	(4,5)
(7,4)	(4,7)
(7,5)	(4,9)
(8,2)	(2,5)
(8,3)	(2,9)
(8,4)	(2,13)
(8,5)	(2,17)
(9,2)	(2,9)
(9,3)	(2,17)
(9,4)	(2,25)

defination Let a_0 be the tested value for chairux coefficient (α, β). Then a function f, is known as chairux iteration function of N, if $f_{a_n}(\alpha, \beta)$= $f_{a_m}(\alpha, \beta)$ $\forall n \neq m$.

defination 2: let $f_{a_0}(\alpha, \beta), f_{a_1}(\alpha, \beta), f_{a_2}(\alpha, \beta), \ldots f_{a_n}(\alpha, \beta), f_{a_{n+1}}(\alpha, \beta) \ldots f_{a_{n+k}}(\alpha, \beta), f_{a_1}(\alpha, \beta) \ldots$

Since $f_{a_n}(\alpha, \beta) = f_{a_{n+k}}(\alpha, \beta)$ then k is known as Chairux iteration period for (α, β)

Illustration

Let $a_0 = 10x_0 + y_0$

then $f_{a_0}(\alpha, \beta) = \alpha x_0 + \beta y_0 = a_1$

$a_i = 10x_i + y_i$

$\therefore f_{a_i}(\alpha, \beta) = \alpha x_i + \beta y_i = a_{i+1}$ $\forall i = 0, 1, 2 \ldots$

Chairux iteration equation

Let $a_0 = 10x_0 + y_0$

$a_1 = \alpha x_0 + \beta y_0$

$\alpha x_i + \beta y_i = a_{i+1}$

$a_1 = \alpha(\frac{(a_0 - y_0)}{10}) + \beta y_0 \implies (\frac{\alpha(a_0-y_0)+10\beta y_0}{10})$

$a_2 = \alpha(\frac{(a_1 - y_1)}{10}) + \beta y_1 \implies (\frac{\alpha(a_1-y_1)+10\beta y_1}{10})$

$a_3 = \alpha(\frac{(a_2 - y_2)}{10}) + \beta y_2 \implies (\frac{\alpha(a_2-y_2)+10\beta y_2}{10})$

$\therefore a_{n+1} = \alpha(\frac{(a_n - y_n)}{10}) + \beta y_n \implies (\frac{\alpha(a_n-y_n)+10\beta y_n}{10})$

Example

find chairux iteration of 4270893 for (1,7):

Solution

$f_{4270893}(1,7) = 427089 \times 1 + 7 \times 3 = 427110$

$f_{427110}(1,7) = 42711 \times 1 + 7 \times 0 = 42711$

$f_{42711}(1,7) = 4271 \times 1 + 7 \times 1 = 4278$

$f_{4278}(1,7) = 427 \times 1 + 7 \times 8 = 483$

$f_{483}(1,7) = 48 \times 1 + 7 \times 3 = 69$

$f_{69}(1,7) = 6 \times 1 + 7 \times 9 = 69$

Defination: The is inverse of a given number for chairux coefficient (α, β) and given by:

$[f_{n_0}(\alpha,\beta)]^-1 = \sum_{k=0}^{n-1} 10^k y_k$ is known as inversion equation of order one.

illustration

$a_1 = \alpha(\frac{(a_0 - y_0)}{10}) + \beta y_0 \implies (\frac{\alpha(a_0-y_0)+10\beta y_0}{10})$

$\therefore a_1 \implies \frac{\alpha^0}{10^1}[\alpha a_0 + y_0(10\beta - \alpha)]$

$a_2 \implies (\frac{\alpha(a_1-y_1)+10\beta y_1}{10}) \implies (\frac{\alpha(\frac{\alpha(a_0-y_0)+10\beta y_0}{10})-y_1)+10\beta y_1}{10})$

$a_2 \implies \frac{1}{10}[\alpha(\frac{\alpha a_0 - \alpha y_0 + 10\beta y_0}{10}) - \alpha y_1 + 10\beta y_1]$

$a_2 \implies [\alpha\frac{\alpha a_0 - \alpha y_0 + 10\beta y_0 - 10\alpha y_1 + 100\beta y_1}{10^2}]$

$\therefore a_2 \implies \frac{\alpha^1}{10^2}[\alpha a_0 + (10\beta - \alpha)(y_0 + 10y_1)]$

$\therefore a_3 \implies \frac{\alpha^2}{10^3}[\alpha a_0 + (10\beta - \alpha)(y_0 + 10y_1 + 100y_2)]$

$\therefore a_n \implies \frac{\alpha^{n-1}}{10^n}[\alpha a_0 + (10\beta - \alpha)(y_0 + 10y_1 + 100y_2 + \cdots + 10^{n-1}y_{n-1})]$

in general chairux inversionl equation is given by:

$$a_n = \frac{\alpha^{n-1}}{10^n}[\alpha a_0 + (10\beta - \alpha)\sum_{k=0}^{n-1} 10^k y_k]$$

$let(10\beta - \alpha)$ be γ then:

$$a_n = \frac{\alpha^{n-1}}{10^n}[\alpha a_0 + \gamma\sum_{k=0}^{n-1} 10^k y_k]$$

$$\therefore \sum_{k=0}^{n-1} 10^k y_k = \frac{\frac{10^n \times a_n}{\alpha^{n-1}} - \alpha a_0}{\gamma}$$

Example

find chairux inversion of $f_{4270893}(1, 7)$.

Solution

Chairux inversion is given by:

$$\sum_{k=0}^{n-1} 10^k y_k = \frac{\frac{10^n \times a_n}{\alpha^{n-1}} - \alpha a_0}{\gamma}$$

$\gamma = 10\beta - \alpha$

$\gamma = 10 \times 7 - 1 = 69$

$\frac{\gamma}{k} \leq N \leq \gamma \; \forall \; k = 1, 2, 3, \ldots$

suppose k=1

N=69

$$[f_{4270893}(1, 7)]^{-1} = \frac{10^5 \times 69 - 4270893}{69} = 38, 103$$

$$[f_{\textbf{4270893}}(1,7)]^{-1} = \frac{10^6\times69-4270893}{69} = 938,103$$

$$[f_{\textbf{4270893}}(1,7)]^{-1} = \frac{10^7\times69-4270893}{69} = 9,938,103$$

$$[f_{\textbf{4270893}}(1,7)]^{-1} = \frac{10^10\times69-4270893}{69} = 9,999,938,103$$

in general

$$[f_{\textbf{4270893}}(1,7)]^{-1} = \frac{10^t\times69-4270893}{69} = 9_{t-5}38,103$$

$\therefore$ chairux inversion=$38,103$

The graph of y_n against n is known Chairux inversion graph.

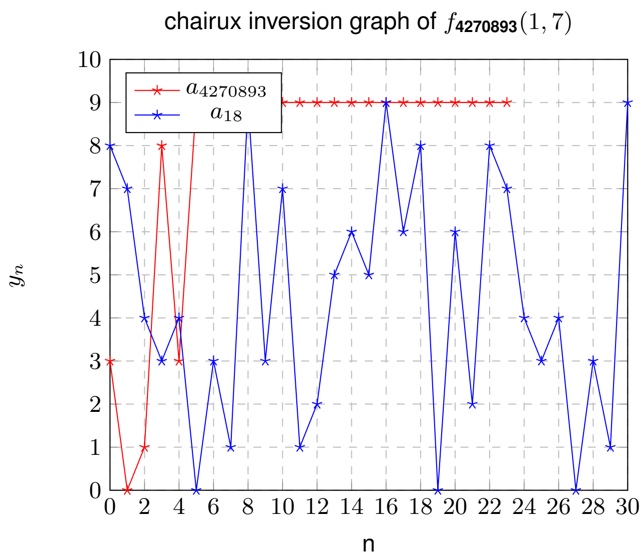

Fig. 1. chairux inversion graph of f4270893(1, 7)

2.5 Chairux period length (λ_l)

Let $f_{a_i}(\alpha,\beta) = a_{i+1}$ be an iteration function for i=0,1,2,3,. . . n,. . . m,. . . if

$f_{a_n}(\alpha,\beta)$=$f_{a_m}(\alpha,\beta)$ then:

$|m-n|$ is known as the period length of (α,β) and denoted as:

λ_l=$|m-n|$

Table 3. Chairux period length (λ_l) table

(α, β)	1	2	3	4	5	6	7	8	9	10
1	01	18	28	06	42	58	22	13	44	02
2	01	06	06	09	04	14	16	04	05	42
3	01	08	01	18	46	09	66	10	14	96
4	01	01	03	03	22	02	10	06	13	01
5	01	04	20	06	06	10	14	20	08	18
6	01	03	02	04	05	01	04	12	03	44
7	01	10	11	05	42	26	03	03	41	15
8	01	02	05	01	06	12	11	02	04	22
9	01	06	06	05	20	01	60	35	01	06

Theorem 2.2. *let $|m-n|$ be chairux period length and a_0 be the tested value of N. Then a_0 is divisible by N if and only if the period length is one.*

i.e λ_l=$|m-n|$=1

Not divisible by N if:

$|m-n|$ >***1***

Example

1) Find wheather the following are divisible by 37.

a) 2181964 c) 5881964

b) 236754321 d) 95421

2) Find wheather the following are divisible by 23.

a) 42348903 c) 7234597

b) 28395041 d) 181574903

3) Find wheather the following are divisible by 84.

a) 81964 c) 4786740

b) 626547684 d) 2157894

Solution

1.a) 37

$N = 10x + y$

$37 = 10 \times 3 + 7$

$y = 7 > 5$

since $y > 5$

$\alpha = 10 - y = 10 - 7 =$ **3**

$\beta = x + 1 = 3 + 1 =$ **4**

$\therefore (\alpha, \beta) =$ **(3,4)**

$f_{2181964}(3, 4) = 3 \times 218196 + 4 \times 4 = 654604$

$f_{654604}(3, 4) = 3 \times 65460 + 4 \times 4 = 196396$

$f_{196396}(3, 4) = 3 \times 19639 + 4 \times 6 = 58941$

$f_{58941}(3, 4) = 3 \times 5894 + 4 \times 1 = 17686$

$f_{17686}(3, 4) = 3 \times 1768 + 4 \times 6 = 5328$

$f_{5328}(3, 4) = 3 \times 532 + 4 \times 8 = 1628$

$f_{1628}(3, 4) = 3 \times 162 + 4 \times 8 = 518$

$f_{518}(3, 4) = 3 \times 51 + 4 \times 8 = 185$

$f_{185}(3, 4) = 3 \times 18 + 4 \times 5 = 74$

$f_{74}(3, 4) = 3 \times 7 + 4 \times 4 = 37$

$f_{37}(3, 4) = 3 \times 3 + 4 \times 7 = 37$

since $f_{74}(3, 4)$=$f_{37}(3, 4)$=37

$\therefore$ 2181964 is divisible by 37.

1.c)

$N = 10x + y$

$37 = 10 \times 3 + 7$

$y = 7 > 5$

since $y > 5$

$\alpha = 10 - y = 10 - 7 =$ **3**

$\beta = x + 1 = 3 + 1 =$ **4**

$\therefore (\alpha, \beta) =$ **(3,4)**

$f_{5881964}(3, 4) = 3 \times 588196 + 4 \times 4 = 1764604$

$f_{1764604}(3, 4) = 3 \times 176460 + 4 \times 4 = 529396$

$f_{529396}(3, 4) = 3 \times 52939 + 4 \times 6 = 158841$

$f_{158841}(3, 4) = 3 \times 15884 + 4 \times 1 = 47656$

$f_{47656}(3, 4) = 3 \times 4765 + 4 \times 6 = 14319$

$f_{14319}(3, 4) = 3 \times 1431 + 4 \times 9 = 4329$

$f_{4329}(3, 4) = 3 \times 432 + 4 \times 9 = 1332$

$f_{1332}(3, 4) = 3 \times 133 + 4 \times 2 = 407$

$f_{407}(3, 4) = 3 \times 40 + 4 \times 7 = 148$

$f_{148}(3, 4) = 3 \times 14 + 4 \times 8 = 74$

$f_{74}(3, 4) = 3 \times 7 + 4 \times 4 = 37$

since $f_{74}(3, 4)$=$f_{37}(3, 4)$=37

$\therefore$ 5881964 is divisible by 37.

2.6 Chairux Iteration Bound

Let $f_N(\alpha, \beta)$ be chairux function of N. Then chairux iteration bound is given by:

$a_i = N \pm k \; \forall 1 \leq k < N$

chairux iteration is either upper or lower

Chairux iteration bound graph

Let $N = 10x + y$ and a_{i+1}=$\alpha x_i + \beta y_i$. Then Chairux differential iteration is given by:

Δ_i=$\frac{a_{i+1}-a_i}{y}$ $\forall$ i=0,1,2,...

A graph of Δ_i against i is known as chairux iteration graph.

The graph of chairux iteration below zero is known as lower bound and above is known as upper bound graph.

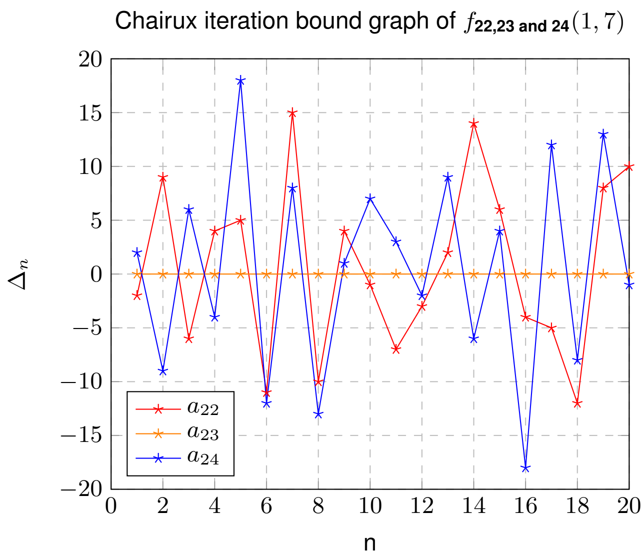

Fig. 2. Chairux iteration bound graph of f$_{22,23and24}(1,7)$

2.7 Chairux Divisor N for (α, β)

This are various method of finding N given chairux coefficient (α, β). This methods include:

1) Structural method

This is the simplest method of finding N given Chairux coefficient. this method is also known as inverse method.

Recall

$\alpha = 10 - ky$, $\beta = 1 + kx$ and $N = 10x + y$

$ky = 10 - \alpha \implies y = \frac{10-\alpha}{k}$

$kx = \beta - 1 \implies x = \frac{\beta-1}{k}$

But we know: $N = 10x + y$

$N = 10(\frac{\beta-1}{k}) + (\frac{10-\alpha}{k})$

$N = \frac{1}{k}(10\beta - 10 + 10 - \alpha)$

$N = \frac{1}{k}(10\beta - \alpha)$

k is equal to gcd $[(10\beta - \alpha), y]$

y is the last digit of $(10\beta - \alpha)$

$\therefore N_{min} = \frac{(10\beta-\alpha)}{gcd[(10\beta-\alpha),y]}$ **of** (α, β)

2) Iteration method

Let a_m be the inverse of a_n and $f_{a_m}(\alpha, \beta) = f_{a_n}(\alpha, \beta)$ s.t $a_m \neq a_n$.

$\therefore N_{min} = \frac{(|a_m - a_n|)}{gcd[(|a_m-a_n|),y]}$ **of** (α, β)

Example

Given that $a_m = 36$ for chairux coefficient $(4, 5)$. Calculate chairux divisor N.

Solution

a_n**=**$(a_m)^{-1}$ **s.t** $a_m \neq a_n$

$f_{m+1}(4, 5) = f_{36}(4, 5) = 3 \times 4 + 5 \times 6 = 42$

$4x + 5y = 42$

$4x = 42 - 5y$

$x = \frac{42-5y}{4}$

x is a whole nuber when y is either 2 or 6

i) when y=2

$x = \frac{42-5\times2}{4} = 8$

$a_n = 10x + y = 82$

ii) when y=6

$x = \frac{42-5\times6}{4} = 3$

$a_n = 10x + y = 36$

since $a_m \neq a_n$

$\therefore a_n = 82$

36**=**$(82)^{-1}$

$f_{82}(4,5) = f_{36}(4,5) = 42$

$|a_m - a_n|$=$|36 - 82|$=46

$\therefore N_{min} = \frac{f46}{gcd(46,6)} = \frac{46}{2} = 23$

3) Bi-section method

Let a_0 and b_0 be 2 ivp s.t $b_0 = ka_0$ for $k > 1$, If the function $f_{a_n}(\alpha,\beta) > f_{b_n}(\alpha,\beta)$, Then Chairux divisor N is given by:

$N = ka_n - b_n$

chairux minimum divisor is given by;

(α,β)	(α,β)
$a_0 = \Omega$	$b_0 = k\Omega$
$f_{a_0}(\alpha,\beta) = a_1$	$f_{b_0}(\alpha,\beta) = b_1$
$f_{a_1}(\alpha,\beta) = a_2$	$f_{b_1}(\alpha,\beta) = b_2$
$\vdots$	$\vdots$
$f_{a_{n-1}}(\alpha,\beta) = a_n$	$f_{b_{n-1}}(\alpha,\beta) = b_n$

$\therefore N_{min} = \frac{(ka_n-b_n)}{gcd[(ka_n-b_n),y]}$ **of** (α,β)

Example

Using bisection method, find chairux minimum divisor (N_{min}) given chairux coefficient $(3,6)$.

Solution

$(3,6)$	$(3,6)$
$a_0 = 10$	$b_0 = 20$
$a_1 = 3\times1+0\times6 = 03$	$b_1 = 3\times2+0\times6 = 06$
$a_2 = 3\times0+3\times6 = 18$	$b_2 = 3\times0+6\times6 = 36$
$a_3 = 3\times1+8\times6 = 51$	$b_3 = 3\times3+6\times6 = 45$
$a_4 = 3\times5+1\times6 = 21$	$b_4 = 3\times4+5\times6 = 42$

since $a_3 > b_3$

$\gamma = 2a_3 - b_3$

$\gamma = 2 \times 51 - 45 = 57$

$N_{min}=\frac{f_\gamma(3,6)}{gcd[f_\gamma(3,6),y]} \implies \frac{f_{57}(3,6)}{gcd[f_{57}(3,6),7]}=\frac{57}{gcd(57,7)}=57$

Chairux order

Let (a,b) be a given pair of Chairux coefficient,Then Chairux order denoted as $\ll N \gg$ is given by:

$\ll N \gg=\gamma^\theta$ gcd($\frac{(a,b)}{[\gamma^\theta,(10-y)]}$)

$\gamma^\theta=\frac{\gamma}{gcd[\gamma,(10-y)]}$

$\gamma = N = ka_n - b_n$

Theorem 2.3. *Let (a,b) be a given pair of chairux coefficient s.t $b > a$. if the function $f_{a_n}(\alpha,\beta) > f_{b_n}(\alpha,\beta)$. Then Chairux order denoted as $\ll N \gg$ is given by:*

$\ll N \gg=\frac{a_n \times b_0 - b_n \times a_0}{gcd[,y]}$

2.8 Chairux Polynomial Mean

The average mean of Chairux identity denoted as $P_n(k)$, whereby n and k are the rate and order of chairux identity respectively is known as Chairux polynomial mean.

Theorem 2.4. *Every chairux identity with the same factor, have the same polynomial mean.*

Proof. since all divisor having the common multiple factor have the same polynomial mean.

Chairux polynomial mean can be illustrated by table.

Table 4. Chairux polynomial mean $P_n(k)$ table

	(n,k)	1	2	3	4	...	...	...	...	...	k
$\alpha = 1$	1	00_01	10_01	20_01	30_01	...	...	...	...	...	$(k-1)0_01$
	2	00_11	10_11	20_11	30_11	...	...	...	...	...	$(k-1)0_11$
	3	00_21	10_21	20_21	30_21	...	...	...	...	...	$(k-1)0_21$
	4	00_31	10_31	20_31	30_31	...	...	...	...	...	$(k-1)0_31$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	n	$00_{n-1}1$	$10_{n-1}1$	$20_{n-1}1$	$30_{n-1}1$	...	...	...	...	...	$(k-1)0_{n-1}1$

$\alpha = 2$

(n,k)	1	2	3	4	...	...	...	...	...	k
1	01_02	11_02	21_02	31_02	...	...	...	...	...	$(k-1)1_02$
2	01_12	11_12	21_12	31_12	...	...	...	...	...	$(k-1)1_12$
3	01_22	11_22	21_22	31_22	...	...	...	...	...	$(k-1)1_22$
4	01_32	11_32	21_32	31_32	...	...	...	...	...	$(k-1)1_32$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$01_{n-1}2$	$11_{n-1}2$	$21_{n-1}2$	$31_{n-1}2$	...	...	...	...	...	$(k-1)1_{n-1}2$

⋮

$\alpha = \Omega$

(n,k)	1	2	3	...	...	k
1	$0(\Omega-1)_0\Omega$	$1(\Omega-1)_0\Omega$	$2(\Omega-1)_0\Omega$	...	...	$(k-1)(\Omega-1)_0\Omega$
2	$0(\Omega-1)_1\Omega$	$1(\Omega-1)_1\Omega$	$2(\Omega-1)_1\Omega$	...	...	$(k-1)(\Omega-1)_1\Omega$
3	$0(\Omega-1)_2\Omega$	$1(\Omega-1)_2\Omega$	$2(\Omega-1)_2\Omega$	...	...	$(k-1)(\Omega-1)_2\Omega$
4	$0(\Omega-1)_3\Omega$	$1(\Omega-1)_3\Omega$	$2(\Omega-1)_3\Omega$	...	...	$(k-1)(\Omega-1)_3\Omega$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$0(\Omega-1)_{n-1}\Omega$	$1(\Omega-1)_{n-1}\Omega$	$2(\Omega-1)_{n-1}\Omega$	...	...	$(k-1)(\Omega-1)_{n-1}\Omega$

$\therefore P_n(k) = (k-1)_1(\Omega-1)_{n-1}\Omega_1$

$P_n(k) = 10^{1+(n-1)}(k-1)_1 + 10^1(\Omega-1)_{n-1} + \Omega_1$

$P_n(k) = 10^n(k-1)_1 + 10^1(\Omega-1)_{n-1} + \Omega_1$

But $10^n = 1_{n+1} - 1_n$, $10_{n-1} = 1_n - 1$ and $A_nB_m = A_mB_n$,from chairux repeatal statement.

$P_n(k) = 10^n(k-1)_1 + 10_{n-1}(\Omega-1) + \Omega_1$

$P_n(k) = 10^n(k-1)_1 + (1_n-1)(\Omega-1) + \Omega_1$

$P_n(k) = 10^n(k-1)_1 + \Omega_n - 1_n - \Omega_1 + 1 + \Omega_1$

$P_n(k) = 10^n(k-1)_1 + (\Omega-1)_n + 1$

$P_n(k) = (1_{n+1} - 1_n)(k-1)_1 + (\Omega-1)_n + 1$

$P_n(k) = k_{n+1} - 1_{n+1} + \Omega_n - k_n + 1$

$P_n(k) = k(10^n + 1_n) - 1_{n+1} - k_n + 1_n + \Omega_n 1_n + 1$

$P_n(k) = 10^n k + k_n - k_n + \Omega_n - (1_{n+1} - 1)$

$P_n(k) = 10^n k + \Omega_n - 10_n$

$\therefore P_n(k) = 10^n k + (\Omega - 10)_n$

2.9 Properties of Chairux Polynomial Mean

1) $P_0(k) = k$

2) $P_n(0) = (\Omega - 10)_n$

3) $P_n(1) = (\Omega - 10)_n + 1 = P_n(0) + 1$

4) $P_1(1) = \Omega$

5) $P_1(k) = 10(k-1) + \Omega$

6) $P_n(k) = 10^n k + (\Omega - 10)_n$

When the rate of chairux polynomial mean countiniuosly increase by one unit is known as chairux unitizol mean.

Recall:

$$P_n(k) = 10^n k + (\Omega - 10)_n \cdots *$$

$P_{n+1}(k) = 10^{n+1} k + (\Omega - 10)_{n+1} = 10^n.10k + (\Omega - 10).1_{n+1}$

but: $1_{n+1} = 10^n + 1_n$

$P_{n+1}(k) = 10^n.10k + (\Omega - 10)(10^n + 1_n) = 10^n.10k + 10^n(\Omega - 10)_n$

$P_{n+1}(k) = 10^n[10k + (\Omega - 10)_1)] + (\Omega - 10)_n$

$P_{n+2}(k) = 10^{n+1}[10k + (\Omega - 10)_1)] + (\Omega - 10)_{n+1}$

$P_{n+2}(k) = 10^n.10[10k + (\Omega - 10)_1)] + 10^n.(\Omega - 10)_1 + (\Omega - 10)_n$

$P_{n+2}(k) = 10^n[[10^2k + 10(\Omega - 10)_1)] + (\Omega - 10)_1] + (\Omega - 10)_n$

$P_{n+2}(k) = 10^n[10^2k + 10(\Omega - 10)_1) + (\Omega - 10)_1] + (\Omega - 10)_n$

$P_{n+2}(k) = 10^n[10^2k + (\Omega - 10)_2] + (\Omega - 10)_n$

$P_{n+3}(k) = 10^{n+1}[10^2k + (\Omega - 10)_2)] + (\Omega - 10)_{n+1}$

$P_{n+3}(k) = 10^n.10[10^2k + (\Omega - 10)_2)] + 10^n.(\Omega - 10)_1 + (\Omega - 10)_n$

$P_{n+3}(k) = 10^n[[10^3k + 10(\Omega - 10)_2)] + (\Omega - 10)_1] + (\Omega - 10)_n$

$P_{n+3}(k) = 10^n[10^3k + 10(\Omega - 10)_2) + (\Omega - 10)_1] + (\Omega - 10)_n$

$P_{n+3}(k) = 10^n[10^3k + (\Omega - 10)_3] + (\Omega - 10)_n$

Given that n is the rate, k is the order and b is the unitizol constant then Chairux unitizol mean is given by:

$\therefore P_{n+b}(k) = 10^n[10^bk + (\Omega - 10)_b] + (\Omega - 10)_n$

Let 'b' be negative

$P_{n-b}(k) = 10^n[10^{-b}k + (\Omega - 10)_{-b}] + (\Omega - 10)_n$

Remark

$X_{-\rho} = \frac{-X_\rho}{10^\rho}$

$(\alpha - 10)_{-\rho} = \frac{-(\alpha-10)_\rho}{10^\rho}$

$P_{n-b}(k) = 10^n[\frac{k}{10^b} - \frac{(\Omega-10)_b}{10^b}] + (\Omega - 10)_n$

$P_{n-b}(k) = \frac{10^n}{10^b}[k - (\Omega - 10)_b] + (\Omega - 10)_n$

$\therefore P_{n-b}(k) = 10^{n-b}[k - (\Omega - 10)_b] + (\Omega - 10)_n$

Theorem 2.5. *let X be a universal set* $\forall a, b \in X and$ *let* $Y = a + b$ *then;*

$(10^X - 1)_Y = (10^Y - 1)_X$

Proof. we know that $X = a^c + a = b^c + b$ and $Y = a + b$

$(10^m)_n + 1_m = 1_{m+n}$

$(10^{(a+b)^c})_{a+b} + 1_{(a+b)^c} = 1_{(a+b)^c+(a+b)} = 1_X$

but $(a+b)^c = a^c - b$

$(10^{(a+b)^c})_{a+b} = (10^{a^c-b})_{a+b} = \frac{(10^{a^c})_{a+b}}{10^b}$

we know that: $1_{(a+b)^c}$=1_{a^c-b}=$\frac{1_{a^c}-1_b}{10^b}$

$\frac{(10^{a^c})_{a+b}}{10^b} + \frac{1_{a^c}-1_b}{10^b} = 1_X$

$(10^{a^c})_{a+b} + 1_{a^c} - 1_b = 10^b{}_X$

$(10^{a^c})_Y + 1_{a^c} - 1_b = 10^b{}_X$

$(10^{X-a})_Y + 1_{X-a} - 1_b = 10^b{}_X$

$\frac{(10^X)_Y}{10^a} + \frac{1_X - 1_a}{10^a} = (10^b)_X + 1_b$

$(10^X)_Y + 1_X - 1_a = 10^a(10^b)_X + (10^a)_b$

$(10^X)_Y + 1_X = 10^a(10^b)_X + (10^a)_b + 1_a$

$(10^X)_Y + 1_X = (10^{a+b})_X + (10^a)_b + 1_a$

$(10^X)_Y + 1_X = (10^Y)_X + (10^a)_b + 1_a$

$(10^X)_Y + 1_X = (10^Y)_X + 1_{a+b}$

$(10^X)_Y + 1_X = (10^Y)_X + 1_Y$

$(10^X)_Y - 1_Y = (10^Y)_X - 1_X$

$\boxed{(10^X - 1)_Y = (10^Y - 1)_X \forall_{X,Y} \in \mathbb{C}}$ □

Theorem 2.6. *let X be a universal set* $\forall a, b \in X and$ *let* $Y = a - b$ *then;*

$(10^X - 1)_Y = (10^Y - 1)_X$

Proof. we know that $X = a^c + a = b^c + b$ and $Y = a - b$

$(10^m)_n + 1_m = 1_{m+n}$

$(10^{(a-b)^c})_{a-b} + 1_{(a-b)^c} = 1_{(a-b)^c+(a-b)} = 1_X$

but $(a-b)^c = a^c + b$

$(10^{a^c+b})_{a-b} + 1_{a^c+b} = 1_X$

$10^b(10^{a^c})_{a-b} + (10^b)_{a^c} + 1_b = 1_X$

$10^b[(10^{a^c})_{a-b} + 1_{a^c}] = 1_X - 1_b$

$10^b[(10^{X-a})_{a-b} + 1_{X-a}] = 1_X - 1_b$

$10^b[\frac{(10^X)_{a-b}}{10^a} + \frac{1_X - 1_a}{10^a}] = 1_X + 1_b$

$10^{b-a}[(10^{X-a})_{a-b} + 1_{X-a}] = 1_X - 1_b$

$10^{-Y}[(10^{X-a})_{a-b} + 1_{X-a}] = 1_X - 1_b$

$[(10^{X-a})_{a-b} + 1_{X-a}] = (10^Y)_X - (10^Y)_b$

$1_a = 1_{Y+b} \implies (10^Y)_b + 1_Y$

$(10^X)_Y + 1_X - ((10^Y)_b + 1_Y) = (10^Y)_X - (10^Y)_b$

$(10^X)_Y - 1_Y = (10^Y)_X - 1_X$

$\boxed{(10^X - 1)_Y = (10^Y - 1)_X \forall_{X,Y} \in \mathbb{C}}$ □

2.10 Chairux Repeatal Function

This is a technique involving arrangement of common series of an integers from statement to column then to row.

The number of times the main repeat itself is known as Repeatal number

Statement which give the main data 'X' and the number of time's it repeat itself (n) is known as repeatal statement and denoted as (X_n).

a) Chairux column: This is arrangement of repeatal statement in a vertical order.

$$X_n = \begin{pmatrix} X \\ X \\ \vdots \\ X \end{pmatrix}$$

b) **Chairux row**: This is arrengement of repeatal statement in a horizontal order.

$$X_n = X(\sum_{k=1}^{n} 10^{k-1}) = Xf_{10}(n) = X\begin{pmatrix}1 & 1 & 1 & \dots & 1\end{pmatrix}$$

Properties of Repeatal Statement

1) $X_n = 10^{n-b}X_b + X_{n-b}$

2) $X_0 = 0$

3) $X_1 = X$

4) $X_n \pm Y_n = (X \pm Y)_n$

5) $X_n \pm Y_m = X_{n-m}10^m + (X \pm Y)_m$

6) $X_n \times Y_m = X_m \times Y_n = (XY)_{n||m} = (XY)_{m||n}$

7) $X_{-b} = -\frac{X_b}{10^{-b}}$

Proof. $X_n = 10^{n-b}X_b + X_{n-b}$

let $b = n$ then:

$X_n = 10^{n-n}X_n + X_{n-n}$

$X_n = 10^0X_n + X_0$

$X_0 = X_n - X_n$

$X_0 = 0$ □

Proof. let $n = 0$ then:

$X_0 = 10^{-b}X_b + X_{-b}$

but $X_0 = 0$

$10^{-b}X_b + X_{-b} = 0$

$X_{-b} = -10^{-b}X_b$

$X_{-b} = -\frac{X_b}{10^{-b}}$ □

Proof. Recall: $X_n = 10^{n-b}X_b + X_{n-b}$

let $n = a + b$ then:

$X_{a+b} = 10^a X_b + X_a$

let $a = a - b$ then:

$X_{a-b+b} = 10^{a-b}X_b + X_{a-b}$

$X_a = 10^{a-b}X_b + X_{a-b}$

$X_{a+b} = 10^b X_a + X_b = 10^a X_b + X_a$

$X_{a+b} = 10^b X_a + X_b$ □

Example

Use repeatal statement to solve:

$a)7_4 \times 5_3 \quad b)5_4 \times 7_3$

Solution

a) $$(7 \times 5)_{4||3} = (35)_{4||3} = (35)_4 = \begin{pmatrix} 3 & 5 & & & \\ & 3 & 5 & & \\ + & & 3 & 5 & \\ & & & 3 & 5 \end{pmatrix} = (38885)_3 = \begin{pmatrix} 3 & 8 & 8 & 8 & 5 & & \\ + & 3 & 8 & 8 & 8 & 5 & \\ & & 3 & 8 & 8 & 8 & 5 \end{pmatrix} = 4316235$$

a) $$(5 \times 7)_{3||4} = (35)_{3||4} = (35)_3 = \begin{pmatrix} 3 & 5 & & \\ + & 3 & 5 & \\ & & 3 & 5 \end{pmatrix} = (3885)_4 = \begin{pmatrix} 3 & 8 & 8 & 5 & & & \\ & 3 & 8 & 8 & 5 & & \\ + & & 3 & 8 & 8 & 5 & \\ & & & 3 & 8 & 8 & 5 \end{pmatrix} = 4316235$$

$\therefore X_n \times Y_m = X_m \times Y_n = (XY)_{n||m} = (XY)_{m||n}$

defination: Chairux repeatal function f_{10} is known as chairux repeatal linear function if it have more than one rate and given as:

$f_{10}(a + b + c + \dots)$

Properties of Repeatal function

1) $f_{10}(0) = 0$

2) $f_{10}(1) = 1$

3) $f_{10}(-n) = \frac{-f_{10}(n)}{10^n}$

4) $f_{10}(a+b) = 10^a f_{10}(b) + f_{10}(a) = 10^b f_{10}(a) + f_{10}(b)$

5) $f_{10}(n) = 1_n = \sum_{k=1}^{n} 10^{k-1}$

Chairux Repeatal Table

This is a table of chairux function of base 10 and rate less to absolute value of one.

When the rate is more than one we use chairux linearity property.

i.e $f_{10}(a+b) = 10^a f_{10}(b) + f_{10}(a) = 10^b f_{10}(a) + f_{10}(b)$

Example

Use repeatal function of a whole number rate to solve:

$a) f_{10}(3) \quad b) f_{10}(1) \quad c) f_{10}(-4) \quad d) f_{10}(0)$

Solution

$a) f_{10}(3) = 1_3 = 111$

$b) f_{10}(1) = 1_1 = 1$

$c) f_{10}(-4) = 1_{-4} = \frac{-f_{10}(4)}{10^4} = \frac{-1_4}{10^4} = -0.1111$

$d) f_{10}(0) = 1_0 = 0$

Table 5. Chairux Repeatal ($f_{10}(n)$) Table

n	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0026	0.0052	0.0079	0.0107	0.0136	0.0165	0.0194	0.0225	0.0256
0.1	0.0288	0.0321	0.0354	0.0388	0.0423	0.0458	0.0495	0.0533	0.0571	0.0610
0.2	0.0650	0.0691	0.0733	0.0776	0.0820	0.0866	0.0912	0.0959	0.1007	0.1057
0.3	0.1106	0.1158	0.1211	0.1265	0.1320	0.1377	0.1435	0.1495	0.1555	0.1618
0.4	0.1680	0.1745	0.1812	0.1880	0.1950	0.2021	0.2094	0.2169	0.2246	0.2324
0.5	0.2403	0.2485	0.2569	0.2655	0.2743	0.2832	0.2924	0.3019	0.3115	0.3213
0.6	0.3312	0.3415	0.3521	0.3629	0.3739	0.3852	0.3968	0.4086	0.4208	0.4332
0.7	0.4458	0.4588	0.4721	0.4857	0.4996	0.5138	0.5284	0.5433	0.5585	0.5742
0.8	0.5900	0.6063	0.6231	0.6402	0.6577	0.6756	0,6939	0.7127	0.7319	0.7516
0.9	0.7715	0.7921	0.8131	0.8347	0.8567	0.8793	0.9023	0.9260	0.9501	0.9747

Example

1) Use repeatal function of a rate less than one from repeatal table to solve:

$a)f_{10}(0.3) \quad b)f_{10}(0.71) \quad c)f_{10}(-0.58) \quad d)f_{10}(0.07)$

2) Use chairux linearity function to show that.

$a)f_{10}(0.7) = f_{10}(0.3 + 0.4) \quad b)f_{10}(-0.71) = f_{10}(0.29 - 1)$

3) Use chairux linearity function to solve.

$a)f_{10}(3.7) \quad b)f_{10}(2.71)$

Solution

1) $a)f_{10}(0.3) = 1_{0.3} = 0.1106$

$b)f_{10}(0.71) = 1_{0.71} = 0.4588$

$c)f_{10}(-0.58) = 1_{-0.58} = \frac{-f_{10}(0.58)}{10^{0.58}} = \frac{-0.3115}{10^{0.58}} = -0.0819$

$d)f_{10}(0.07) = 1_{0.07} = 0.0194$

2) $a)f_{10}(0.7) = f_{10}(0.3 + 0.4)$

$f_{10}(0.7) = 0.4458$

$f_{10}(0.3 + 0.4) = 10^{0.3}f_{10}(0.4) + f_{10}(0.3) = 10^{0.3}(0.1680) + 0.1106 = 0.4458$

$f_{10}(0.3 + 0.4) = 10^{0.4}f_{10}(0.3) + f_{10}(0.4) = 10^{0.4}(0.1106) + 0.1680 = 0.4458$

$\therefore f_{10}(0.7) = f_{10}(0.3 + 0.4)$

$b)f_{10}(-0.71) = f_{10}(0.29 - 1)$

$f_{10}(-0.71) = \frac{-f_{10}(0.71)}{10^{0.71}} = \frac{-0.4588}{10^{0.71}} = -0.0895$

$f_{10}(0.29 - 1) = 10^{0.29}f_{10}(-1) + f_{10}(0.29) = 10^{0.29}(-0.1) + 0.1057 = -0.0893$

$f_{10}(0.29 - 1) = 10^{-1}f_{10}(0.29) + f_{10}(-1) = 10^{-1}(0.1057) + -0.1 = -0.0894$

$\therefore f_{10}(-0.71) = f_{10}(0.29 - 1)$

3) $a)f_{10}(3.7) = f_{10}(3+0.7) = f_{10}(4-0.3)$

$f_{10}(3.7) = f_{10}(3+0.7) = 10^3 f_{10}(0.7) + f_{10}(3) = 1000(0.4458) + 111 = 556.8$

$f_{10}(4-0.3) = 10^{-0.3} f_{10}(4) + f_{10}(-0.3) = 0.5012(1111) + -0.0554 = 556.7778$

$\therefore f_{10}(3.7) = 556.8$

Example

Find the value of p in $3_p = 1000$

Solution

$3_p = 3(1_p) = 1000$

$3(1_p) = 1000$

$1_p = \frac{1000}{3} = 333.3\ldots$

$1_3 \leq 333.33 \leq 1_4$

method (i)

$1_{3+k} = 333.33333 \;\; s.t\; 3+k = p$

$1_{3+k} = 10^3(f_{10}(k)) + f_{10}(3) = 1000(f_{10}(k)) + 111 = 333.3\ldots$

$1_{3+k} = 1000(f_{10}(k)) = 333.3\cdots - 111 = 222.3\ldots$

$1_k = \frac{222.3\ldots}{1000} = 0.2223\ldots$

$k = (1_k)^{-1} = (0.2223\ldots)^{-1} = 0.48$

$p = 3 + k = 3 + 0.48 = 3.48$

method (ii)

$1_{4-k} = 333.33333 \;\; s.t\; 4-k = p$

$1_{4-k} = 10^4(f_{10}(-k)) + f_{10}(4) = 10000(f_{10}(-k)) + 1111 = 333.3\ldots$

$1_{4-k} = 10000(f_{10}(-k)) = 333.3\cdots - 1111 = -777.6\ldots$

$1_{-k} = \frac{-777.6\ldots}{1000} = -0.07776\ldots$

$k = (1_{-k})^{-1} = (-0.07776\ldots)^{-1} =$ STUNK!!!

Since this is difficult to solve i provide a negative repeatal table.

Table 6. Chairux Repeatal ($f_{10}(-n)$) Table

$-n$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0025	0.0050	0.0074	0.0098	0.0121	0.0144	0.0165	0.0187	0.0208
0.1	0.0229	0.0249	0.0269	0.0288	0.0306	0.0324	0.0342	0.0360	0.0377	0.0394
0.2	0.0410	0.0426	0.0442	0.0457	0.0472	0.0487	0.0501	0.0515	0.0528	0.0542
0.3	0.0554	0.0567	0.0580	0.0592	0.0603	0.0615	0.0626	0.0638	0.0648	0.0659
0.4	0.0669	0.0679	0.0689	0.0698	0.0708	0.0717	0.0726	0.0735	0.0744	0.0752
0.5	0.0760	0.0768	0.0776	0.0784	0.0791	0.0798	0.0805	0.0813	0.0819	0.0826
0.6	0.0832	0.0838	0.0845	0.0851	0.0857	0.0862	0.0868	0.0874	0.0879	0.0884
0.7	0.0889	0.0895	0.0900	0.0904	0.0909	0.0914	0.0918	0.0923	0.0927	0.0931
0.8	0.0935	0.0939	0.0943	0.0947	0.0951	0.0954	0.0958	0.0961	0.0965	0.0968
0.9	0.0971	0.0974	0.0978	0.0981	0.0984	0.0987	0.0989	0.0992	0.0995	0.0997

$k = (1_{-k})^{-1} = (-0.07776\ldots)^{-1} = -0.53$

$p = 4 - k = 4 - 0.53 = 3.47$

Application Question

1) An equation of diffusion of a certain gas is given as K_{2+c}.

i) Write chairux equation of K_{2+c}.

ii) If k is the rate of diffusion and given by 0.7 iro's.

Find the value of 'c' if a mass of 10g is released with a maximum catalyst of c gram.

Solution

i) $K_{2+c} = k_2 \times 10^c + k_c$

OR

$K_{2+c} = k_c \times 10^2 + k_2$

ii) $k = 0.7$ and $k_{2+c} = 10g$

$0.7_c \times 100 + 0.7_2 = 10$

$0.7_c \times 100 + 7.7 = 10$

$0.7_c \times 100 = 10 - 7.7$

$0.7_c \times 100 = 2.3$

$0.7_c = \frac{2.3}{100} = 0.023$

$0.7 \times 1_c = 0.023$

$1_c = \frac{0.023}{0.7} = 0.0329$

$c = 0.0329_c = 0.11$

3 CONCLUSION

The Chairux algorithm proposed helps us find the divisibility test of any integer. It also helps us find the periodic length of a given integer, which is equal to the period length of the reciprocal of integers. It can also be used to determine the repeated function of any data and also prove the geometric series, whereby $f_z(a+b) = z^a f_z(b) + f_z(a) = z^b f_z(a) + f_z(b) s.t$ $z = x + iy$. Future studies could consider cases where $f_z(a) = \frac{z^a - 1}{z - 1}$.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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