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# Joint Optimal Policy for Variable Deteriorating Inventory System of Vendor-Buyer Ordering with Trapezoidal Demand

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***Authors' contributions***

*This work was carried out in collaboration between all authors. Author NHS proposed the concept of this analysis and author DBS formulated and validated the problem using mathematical tools and Maple software. All authors read and approved the final manuscript.*

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## ABSTRACT

A joint optimal policy for the vendor and the buyer is analyzed when units in inventory are subject to deterioration and demand is trapezoidal. It is shown numerically that the joint venture reduces the total joint cost significantly when compared with the independent decision of both the players. To entice the buyer to place orders of larger size, a permissible credit period is offered by the vendor to the buyer. A negotiation factor is incorporated to share the benefits of cost savings.

*Keywords: Vendor-buyer joint decision; deterioration; credit period; trapezoidal demand.*

## 1. INTRODUCTION

Silver and Meal [1], Silver [2], Xu and Wang [3], Chung and Ting [4,5], Bose et al. [6], Hariga [7], Giri and Chaudhari [8], Lin et al. [9] etc. discussed optimal ordering policy when demand is linearly changing with respect to time which is superficial in the market of fashion goods, air seats, smart phones etc. Mehta and Shah [10] assumed the demand to be exponential time varying which is again unrealistic for a newly launched product. Shah et al. [11] introduced the quadratic demand which is again not observed in the market for an indefinite

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period. In order to have an alternative demand pattern, the trapezoidal demand is considered. This type of demand increases for some time then gets constant up to some time and afterwards decreases exponentially with time.

Most of the models available in the literature assumed that the buyer is the dominant player to make the decision for procurement. This strategy may not be economical for the vendor. An integrated vendor-buyer policy should be analyzed which is beneficial to the players in the supply chain. Clark and Scarf [12] and thereafter, Goyal [13] proposed a mathematical model for vendor-buyer integration. Banerjee [14] discussed an economic lot-size model when production is finite. Cárdenas-Barrón et al. [15] derived vendor-buyer integrated inventory system with arithmetic-geometric inequality. Teng et al. [16] derived vendor-buyer integrated system without derivatives.

Deterioration is defined as the decay, spoilage, evaporation and loss of utility of a product from the original one. Fruits and vegetables, cosmetics and medicines, electronic items, blood components, radioactive chemicals, agriculture products are some of the examples of deteriorating commodities. For articles on deteriorating inventory one can refer to Raafat [17], Shah and Shah [18], Shah et al. [19] and Goyal and Giri [20]. Yang and Wee [21] derived a win – win strategy for an integrated system of vendor-buyer when units in inventory are subject to a constant rate of deterioration and deterministic constant demand. Shah et al. [22] extended above model by incorporating salvage value to the deteriorated units.

In this study, a joint vendor – buyer inventory system is analyzed when demand is trapezoidal and units in inventory are subject to deterioration. A negotiation factor is incorporated to share the savings. The credit period is offered to the buyer to attract the buyer for placing a larger order. A numerical example is illustrated to support the proposed model. Sensitivity analysis is carried out to visualize the changes in cost savings.

## 2. METHODOLOGY

### 2.1 Notations

The proposed study uses following notations.

- $A_b$ Buyer's ordering cost per order
- $A_v$ Vendor's ordering cost per order
- $C_b$ Buyer's purchase cost per unit
- $C_v$ Vendor's purchase cost per unit
- $I_b$ Inventory carrying charge fraction per unit per annum for the buyer
- $I_v$ Inventory carrying charge fraction per unit per annum for the vendor
- $\theta_b$ Deterioration of items in buyer's inventory system
- $\theta_v$ Deterioration of items in vendor's inventory system; $0 < \theta_b < \theta_v < 1$
- $I_b(t)$ Buyer's inventory level at any instant of time t
- $I_v(t)$ Vendor's inventory level at any instant of time t

| | |
|---|---|
| $n$ | Number of orders during cycle time for the buyer (a decision variable) |
| $K_b$ | Buyer's total cost per unit time |
| $K_v$ | Vendor's total cost per unit time |
| $K_{NJ}$ | Total cost of vendor-buyer inventory system when they take independent decision |
| $K_J$ | Total cost of vendor-buyer inventory system when they take joint decision |
| $T$ | Vendor's cycle time (a decision variable) |
| $T_b$ | $\left(=T/n\right)$, Buyer's cycle time (a decision variable) |
| $M$ | Credit period offered by the vendor to the buyer (a decision variable) |
| $r$ | Continuous discounting rate |

## 2.2 Assumptions

The proposed study uses following assumptions.

- A supply chain of a single vendor and single buyer is considered.
- An inventory system deals with a single item.
- The deterioration rates of items in the vendor's and buyer's inventory are different and proportional to on hand stock in inventory. There is no repair or replacement of deteriorated units during a cycle time.
- The demand rate is trapezoidal. Its functional form is

$$R(t)=\begin{cases} f(t) & ;0\le t\le u_1 \\ D_0 & ;u_1\le t\le u_2 \\ g(t) & ;u_2\le t\le T \end{cases}$$

where; $f(t)$ is linear in $t$, $D_0=f(u_1)=g(u_2)$ and $g(t)$ is exponentially decreasing in $t$ (say)

$$R(t)=\begin{cases} a(1+b_1t) & ;0\le t\le u_1 \\ a(1+b_1u_1) & ;u_1\le t\le u_2 \\ a(1+b_1u_1)e^{-b_2(-u_2+t)} & ;u_2\le t\le T \end{cases}$$

where; $a$ denotes scale demand, $0<b_1,b_2<1$ denotes rates of change of demand. (Cheng et al.) [23] (Fig. 1)

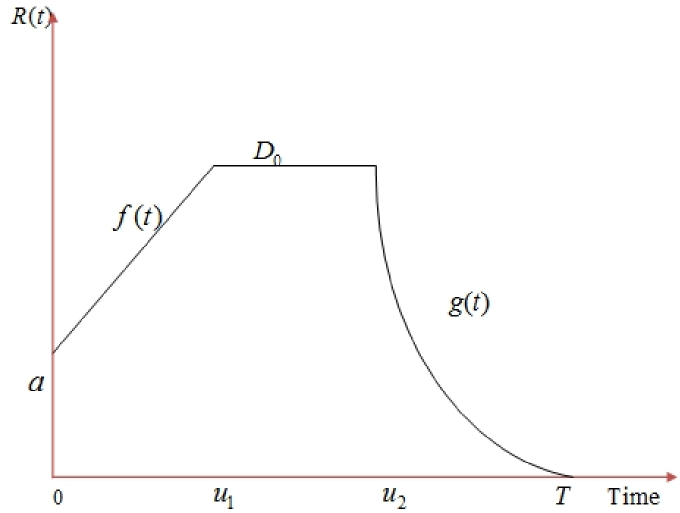


**Fig. 1. Trapezoidal demand**

- The lead time is zero and shortages are not allowed.
- The credit period is offered for settling the accounts due against purchases to attract the buyer to opt a joint decision policy.

## 2.3 Mathematical Model

Fig. 2 depicts the time-varying inventory status for the vendor and the buyer.

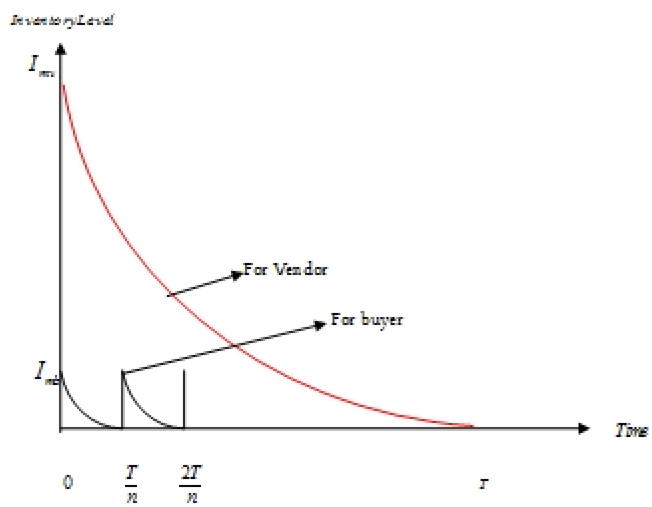


**Fig. 2. Vendor–buyer inventory status**

The inventory changes due to trapezoidal demand for both vendor and buyer. The rate of change of inventory for both the players is governed by the differential equations:

$$\frac{dI_b(t)}{dt} + \theta_b I_b(t) = -R(t), 0 \le t \le T_b \tag{1}$$

$$\frac{dI_v(t)}{dt} + \theta_v I_v(t) = -R(t), 0 \le t \le T \tag{2}$$

with the boundary conditions $I_b(T_b) = 0$, $I_v(T) = 0$ and initial conditions $I_b(0) = I_{mb}$, $I_v(0) = I_{mv}$.

The solutions of the differential equations are

$$I_b(t) = \begin{cases} a\left[-\frac{1+b_1 t}{\theta_b} + \frac{b_1}{\theta_b^{\,2}} + \frac{1+b_1 u_1}{\theta_b} e^{\theta_b u_1 - \theta_b t} - \frac{b_1}{\theta_b^{\,2}} e^{\theta_b u_1 - \theta_b t}\right] + \\ \frac{a(1+b_1 u_1)}{\theta_b}\left[e^{\theta_b u_2 - \theta_b t} - e^{\theta_b u_1 - \theta_b t}\right] + \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_b - b_2}\right]\left[e^{-\frac{b_2 T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2 u_2 + \theta_b u_2 - \theta_b t}\right] & ;0 \le t \le u_1 \\ \\ \frac{a(1+b_1 u_1)}{\theta_b}\left[-1 + e^{\theta_b u_2 - \theta_b t}\right] + \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_b - b_2}\right]\left[e^{-\frac{b_2 T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2 u_2 + \theta_b u_2 - \theta_b t}\right] & ;u_1 \le t \le u_2 \\ \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_b - b_2}\right]\left[e^{-\frac{b_2 T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2 t}\right] & ;u_2 \le t \le T_b \end{cases} \tag{3}$$

$$I_v(t) = \begin{cases} a\left[-\frac{1+b_1 t}{\theta_v} + \frac{b_1}{\theta_v^{\,2}} + \frac{1+b_1 u_1}{\theta_v} e^{\theta_v u_1 - \theta_v t} - \frac{b_1}{\theta_v^{\,2}} e^{\theta v u_1 - \theta_v t}\right] + \\ \frac{a(1+b_1 u_1)}{\theta_v}\left[e^{\theta_v u_2 - \theta_v t} - e^{\theta_v u_1 - \theta_v t}\right] + \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_v - b_2}\right]\left[e^{-b_2 T + \theta_v T - \theta_v t} - e^{-b_2 u_2 + \theta_v u_2 - \theta_v t}\right] & ;0 \le t \le u_1 \\ \\ \frac{a(1+b_1 u_1)}{\theta_v}\left[-1 + e^{\theta_v u_2 - \theta_v t}\right] + \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_v - b_2}\right]\left[e^{-b_2 T + \theta_v T - \theta_v t} - e^{-b_2 u_2 + \theta_v u_2 - \theta_v t}\right] & ;u_1 \le t \le u_2 \\ \\ \left[\frac{a(1+b_1 u_1)e^{b_2 u_2}}{\theta_v - b_2}\right]\left[e^{-b_2 T + \theta_v T - \theta_v t} - e^{-b_2 t}\right] & ;u_2 \le t \le T \end{cases} \tag{4}$$

Using $I_b(0)=I_{mb}$, $I_v(0)=I_{mv}$, the maximum procurement quantities for the buyer and the vendor are

$$I_{mb}=\left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_b-b_2}\right]\left[e^{-\frac{b_2T}{n}+\frac{\theta_bT}{n}}-1\right]$$

$$I_{mv}=\left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_v-b_2}\right]\left[e^{-b_2T+\theta_vT}-1\right]$$

respectively.

During the cycle time $\left[0,T\right]$, for $n$ -shipments, the buyer's

- Purchase Cost; $PC_b=nC_bI_{mb}$
- Holding Cost; $HC_b=nC_bI_b\int_0^{T_b}I_b(t)dt=nC_bI_b\left[\int_0^{u_1}I_b(t)dt+\int_{u_1}^{u_2}I_b(t)dt+\int_{u_2}^{T_b}I_b(t)dt\right]$
- Ordering Cost; $OC_b=nA_b$

Hence, the buyer's total cost; $K_b$ per unit time is

$$K_b=\frac{1}{T}\left[PC_b+HC_b+OC_b\right] \tag{5}$$

The vendor's inventory is the difference between the vendor-buyer combined inventory and the buyer's inventory during n-orders. This is known as the joint two-echelon inventory model. The vendor's

- Purchase Cost; $PC_v=C_vI_{mv}$
- Holding Cost; $HC_v=C_vI_v\left[\int_0^{T}I_v(t)dt-n\int_0^{T_b}I_b(t)dt\right]$
- Ordering Cost; $OC_v=A_v$

Hence, the buyer's total cost; $K_v$ per unit time is

$$K_v=\frac{1}{T}\left[PC_v+HC_v+OC_v\right] \tag{6}$$

The joint total cost $K$ is the sum of $K_b$ and $K_v$ where $T_b=\frac{T}{n}$.

Thus, $K$ is the function of discrete variable $n$ and continuous variable $T$ .

## 2.4 Computational Procedure

There are two cases to be analyzed.

**Case 1:** When the vendor and the buyer take decision independently.

For given value of $n$, differentiate $K_b$ with respect to $T_b$ (equivalently, $T$) and solve $\frac{\partial K_b}{\partial T_b}=0$. This $n$ and $T_b$ minimizes $K_v$ provided

$$K_v(n-1)\geq K_v(n)\leq K_v(n+1) \tag{7}$$

satisfies.
Here, the total cost per unit time with independent decision; $K_{NJ}$ is given by

$$K_{NJ}=\left[\min_n K_b+K_v\right] \tag{8}$$

**Case 2**: When vendor and buyer make decision jointly.
The optimum value of $T$ and $n$ must satisfy the following conditions simultaneously:

$$\frac{\partial K}{\partial T}=0 \text{ (here; } K=K_J\text{)} \quad \text{and} \quad K(n-1)\geq K(n)\leq K(n+1) \tag{9}$$

Thus, the total joint cost is

$$K_J=\min_{n,T}\left[K_b+K_v\right] \tag{10}$$

It is obvious that $K_J \leq K_{NJ}$. Hence, total cost savings $Sav_J$ is defined as $Sav_J=K_{NJ}-K_J$. Now define buyer's cost saving, $Sav_b$ as $Sav_b=\alpha\, Sav_J$, where $0\leq\alpha\leq 1$ is the negotiation factor. When negotiation factor equals to 0.5, saving gets equally distributed between two players; when it is equal to zero, all saving is in the vendor's pocket and when it is equal to 1, it is in favor of the buyer.
The present value of the unit after a time interval M is $e^{-rM}$, where $r$ is discounting rate. Solving the following equation (expression given in Yang and Wee[22])

$$R(t)C_b(1-e^{-rM})=Sav_b \tag{11}$$

the buyer's credit period is given by

$$M=\frac{1}{r}\ln\left[\frac{C_bR(t)}{C_bR(t)-Sav_b}\right] \tag{12}$$

## 3. RESULTS AND DISCUSSION

### 3.1 Numerical Example

Consider following inventory parameters values in proper units:

| $a$ | $b_1$ | $b_2$ | $A_b$ | $A_v$ | $C_b$ | $C_v$ | $I_b$ | $I_v$ | $\theta_b$ | $\theta_v$ | $r$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40000 | 0.04 | 0.02 | 600 | 3000 | 10 | 6 | 0. 11 | 0.10 | .05 | .08 | 0.06 |

Let $\alpha$ = 0.5.
The optimal solution is listed in Table 1 for independent and joint decisions.

**Table 1. Optimal Solution for Independent and Joint Decisions**

| | Case 1 Independent Decision | Case 2 Joint Decision |
|---|---|---|
| $n$ | 3 | 2 |
| $T_b$ | 0.143044678 | 0.193808209 |
| $T$ | 0.429134035 | 0.387616417 |
| $K_b$ | 409026 | 409400 |
| $K_v$ | 254143 | 253467 |
| $K(=K_{NJ} \text{ OR } K_J)$ | $K_{NJ}$ = 663169 | $K_J$ = 662867 |
| $PJCR$ | - | 0.045559667 |
| $M(days)$ | | 5.936896912 |

*(where; $PJCR$ = Percentage change in Joint Cost Reduction)*

The buyer's cost and cycle time increase in joint decisions. The vendor gains $684 and the buyer loses $352. This hinders the buyer to agree for joint decision. To entice the buyer to joint decision, the vendor offers the buyer a credit period of 6.45 days with equal sharing of cost savings. This reduces the joint total cost PJCR by 0.050075188 %, where PJCR is defined as $\frac{K_{NJ}-K_J}{K_J}\times 100$.

The convexity of total integrated cost and independent costs are shown in Fig. 3.

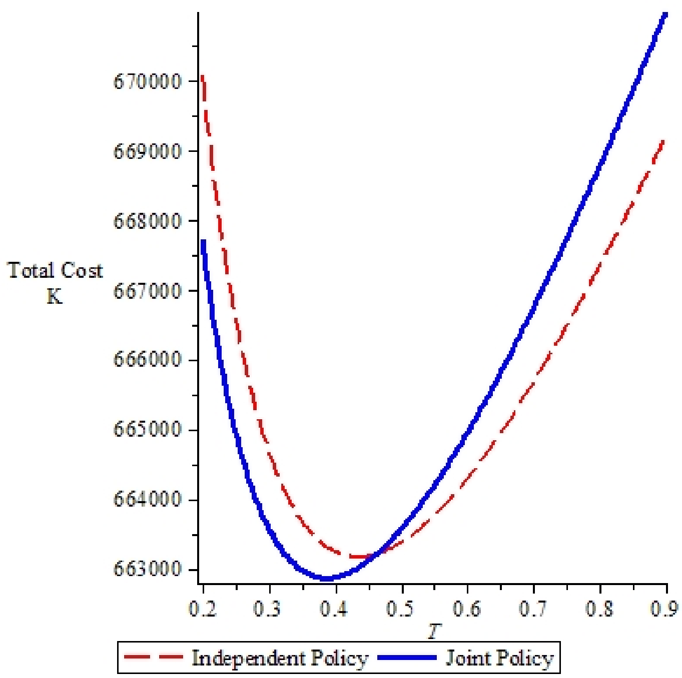


**Fig. 3. Total Cost for Independent Vs Joint Vendor-Buyer Inventory System**

### 3.2 Sensitivity Analysis

**Table 2. Sensitivity analysis of demand rate**

| $a$ | **24000** | **32000** | **40000** | **48000** | **56000** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 401747 | 532616 | 663169 | 793496 | 923650 |
| $K_J$ | 401500 | 532339 | 662867 | 793174 | 923311 |
| $PJCR$ | 0.061519303 | 0.052034512 | 0.045559667 | 0.040596389 | 0.03671569 |
| $M\,(days)$ | 6.265032496 | 6.085058307 | 5.936896912 | 5.783063486 | 5.642417923 |

**Observations**

- Increase in fixed demand $a$, decreases percentage of cost reduction and delay period (Figs. 4 and 5).

**Table 3. Sensitivity analysis of linear rate of change of demand**

| $b_1$ | **0.024** | **0.032** | **0.04** | **0.048** | **0.056** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 662903 | 663036 | 663169 | 663302 | 663435 |
| $K_J$ | 662598 | 662732 | 662867 | 663002 | 663137 |
| $PJCR$ | 0.046030927 | 0.045870729 | 0.045559667 | 0.045248732 | 0.044937924 |
| $M\,(days)$ | 5.9928032 | 5.974688823 | 5.936896912 | 5.8990857 | 5.861255173 |

**Observations**

- Increase in linear rate of change of demand $b_1$, decreases percentage of cost reduction and delay period (Figs. 4 and 5).

**Table 4. Sensitivity analysis of exponential rate of change of demand**

| $b_2$ | **0.012** | **0.016** | **0.02** | **0.024** | **0.028** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 663537 | 663355 | 663169 | 662979 | 662784 |
| $K_J$ | 663253 | 663062 | 662867 | 662666 | 662461 |
| $PJCR$ | 0.042819256 | 0.04418893 | 0.045559667 | 0.047233448 | 0.048757587 |
| $M\,(days)$ | 5.746285797 | 5.84526398 | 5.936896912 | 6.060418484 | 6.156169148 |

**Observations**

- Increase in exponential rate of change of demand $b_2$, increases percentage of cost reduction and delay period (Figs. 4 and 5).

**Table 5. Sensitivity analysis of buyer's ordering cost**

| $A_b$ | **360** | **480** | **600** | **720** | **840** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 661851 | 662410 | 663169 | 664001 | 664857 |
| $K_J$ | 661591 | 662239 | 662867 | 663478 | 664072 |
| $PJCR$ | 0.039299204 | 0.025821493 | 0.045559667 | 0.07882703 | 0.118210074 |
| $M\,(days)$ | 5.433600508 | 3.462024325 | 5.936896912 | 10.00125441 | 14.62485998 |

**Observations**

- Increase in buyer's ordering cost $A_b$, does not reflect the monotonous change in percentage of cost reduction and delay period (Figs. 4 and 5).

**Table 6. Sensitivity analysis of vendor's ordering cost**

| $A_v$ | **1800** | **2400** | **3000** | **3600** | **4200** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 660372 | 661771 | 663169 | 664567 | 665965 |
| $K_J$ | 659508 | 661259 | 662867 | 664363 | 665767 |
| $PJCR$ | 0.13100675 | 0.077428058 | 0.045559667 | 0.030706105 | 0.029740134 |
| $M\,(days)$ | 20.15741179 | 10.88344386 | 5.936896912 | 3.749267486 | 3.430050694 |

**Observations**

- Increase in vendor's ordering cost $A_v$, decreases the percentage of cost reduction and delay period (Figs. 4 and 5).

**Table 7. Sensitivity analysis of buyer's purchase cost**

| $C_b$ | **6** | **8** | **10** | **12** | **14** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 501416 | 582211 | 663169 | 744187 | 825222 |
| $K_J$ | 500216 | 581576 | 662867 | 744100 | 825283 |
| $PJCR$ | 0.239896365 | 0.10918607 | 0.045559667 | 0.011691977 | -0.0073914 |
| $M\,(days)$ | 35.11110597 | 14.7853574 | 5.936896912 | 1.49696577 | -0.94105604 |

**Observations**

- Increase in buyer's purchase cost $C_b$, decreases the percentage of cost reduction and delay period significantly (Figs. 4 and 5).

**Table 8. Sensitivity analysis of vendor's purchase cost**

| $C_v$ | **3.6** | **4.8** | **6** | **7.2** | **8.4** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 564308 | 613738 | 663169 | 712599 | 762030 |
| $K_J$ | 564461 | 613694 | 662867 | 711988 | 761064 |
| $PJCR$ | -0.0271055 | 0.007169697 | 0.045559667 | 0.085816053 | 0.126927565 |
| $M\,(days)$ | -2.69597802 | 0.821269973 | 5.936896912 | 12.59142021 | 20.79023594 |

**Observations**

- Increase in vendor's purchase cost $C_v$, increases the percentage of cost reduction and delay period significantly (Figs. 4 and 5).

**Table 9. Sensitivity analysis of inventory carrying charge fraction of buyer**

| $I_b$ | **0.066** | **0.088** | **0.11** | **0.132** | **0.154** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 661975 | 662535 | 663169 | 663836 | 664515 |
| $K_J$ | 661082 | 661994 | 662867 | 663706 | 664514 |
| $PJCR$ | 0.135081578 | 0.081722795 | 0.045559667 | 0.019586986 | 0.000150486 |
| $M\,(days)$ | 16.11164532 | 10.20714328 | 5.936896912 | 2.654560347 | 0.021155042 |

**Observations**

- Increase in buyer's inventory carrying charge fraction $I_b$, decreases the percentage of cost reduction and delay period significantly (Figs. 4 and 5).

**Table 10. Sensitivity analysis of inventory carrying charge fraction of vendor**

| $I_v$ | **0.06** | **0.08** | 0.1 | **0.12** | **0.14** |
|---|---|---|---|---|---|
| $K_{NJ}$ | 661780 | 662475 | 663169 | 663863 | 664557 |
| $K_J$ | 661903 | 662391 | 662867 | 663333 | 663789 |
| $PJCR$ | -0.01858278 | 0.012681332 | 0.045559667 | 0.079899538 | 0.115699417 |
| $M\,(days)$ | -2.30712429 | 1.613830314 | 5.936896912 | 10.65083491 | 15.76376837 |

**Observations**

- Increase in vendor's inventory carrying charge fraction $I_v$, increases the percentage of cost reduction and delay period significantly (Figs. 4 and 5).

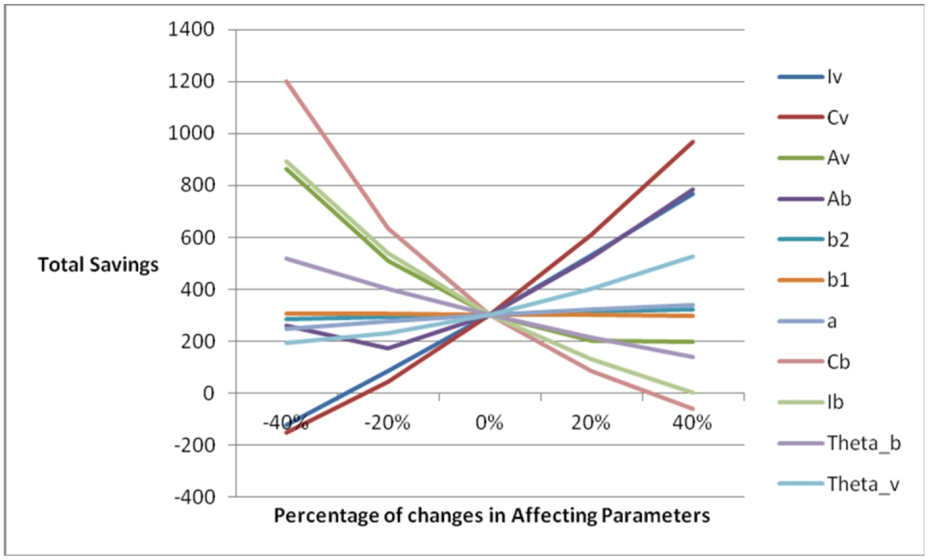


**Fig. 4. Total savings vs. percentage of changes in affecting parameters**

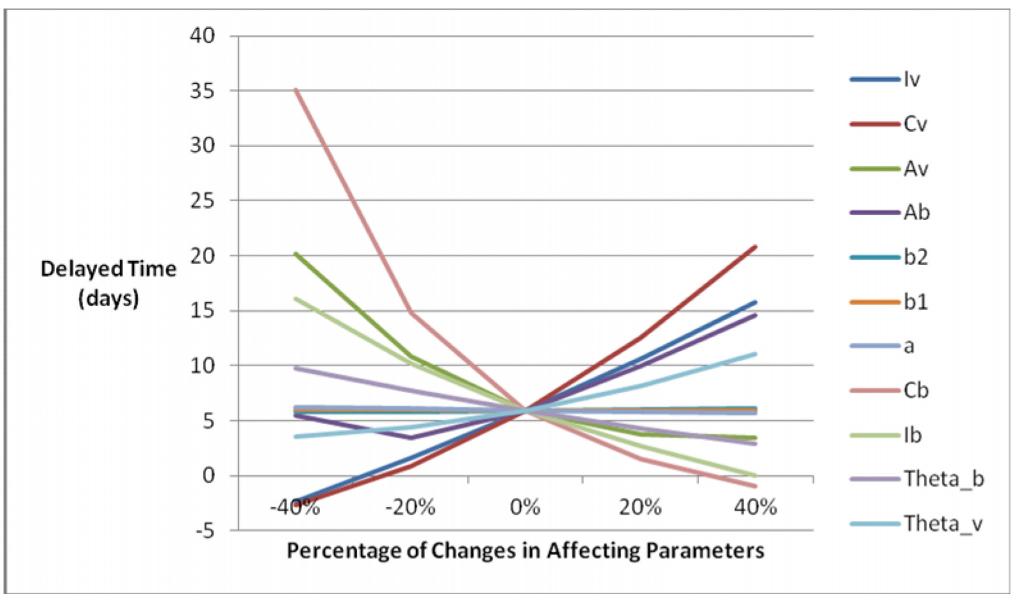


**Fig. 5. Delayed time in days vs. percentage of changes in affecting parameters**

## 4. CONCLUSION

In this paper, a mathematical model is developed to analyze an optimal ordering policy for a supply chain comprising of vendor-buyer inventory system when demand is trapezoidal. The deterioration rate of units in vendor-buyer inventory system is considered to be different. It is established that the joint decision lowers the total cost of an inventory system, even though the buyer's cost increases significantly. To attract the buyer for a joint decision, a credit period is offered by the vendor to the buyer to settle the account. The problem can be studied for time-dependent deterioration. One can apply the DCF approach to visualize financial perspectives of the goals.

Vendor's purchase cost, inventory carrying charge fraction and deterioration of units in the inventory system has positive impact on the delay period and total cost savings. The first two parameters are uncontrollable but we can keep a check on deterioration rate of items by using necessary storage facilities. The ordering cost of buyer has positive impact on total cost savings and delay period but it is uncontrollable because of fluctuations in fuel price.

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## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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