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Solving Investment Problem with Inexact Rough Interval Data through Dynamic Programming Approach

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Abstract

Finding the best investment is an interesting optimization problem. When we deal with such a problem in an uncertain and vague environment, the optimization problem becomes more difficult. In this paper, we deal with rough data expressed in the form of an inexact rough interval fuzzy numbers, and then solve the optimization problem for atypical investment problem to obtain a rough interval solution. The process of optimization is illustrated by a numerical example.

Keywords: Investment problem; rough interval; optimal rough interval solution; dynamic programming.

1 Introduction

The world is becoming more and more a global market place and the global environment is forcing companies to take almost everything into consideration at the same time.

The investment optimization problem in a directed a cyclic graph has been widely applied in practice such as project managements. Usually, some problems concerned by a decision maker (DM) are whether the project would finish before a given deadline and how we should invest capital provided that qualities of the project may not be under the normal level. For cost is one of

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the most important factors the decision maker concerned, the investment optimization appears to hold the balance in the real project management.

Abel and Eberly [1] unified an investment model under uncertainty in a dynamic programming problem. Lee and Shin [2] studied two types of fixed costs: The first assumes a lump-sum cost that has to be paid to set up a project and the second assumes fixed costs per unit time that are independent of the level of investment and are incurred at each point in time when investment is non-zero. Kahraman et al. [3] applied dynamic programming to the situation where each investment in the set has several possible values, and the rate of return varies with the amount invested. Ammar and Khalifa [4] characterized the optimal solutions of uncertainty investment problem with trapezoidal fuzzy numbers. Cooper and Haitiwanger [5] proved that the fixed costs are modeled proportional to capital stock. Tahar et al. [6] introduced an extension of the Merton optimal investment problem to the case where the risk asset is subject to transaction costs and capital gains taxes. They derived the dynamic programming equation in the sense of constrained viscosity solutions. Modarres et al. [7] used a dynamic programming approach to obtain the optimal policies for an investor who faces as stochastic number of investing chances (with Poisson distribution) and a stochastic profit for every chance accruing (with uniform distribution). Xu [8] developed a new two-stage fuzzy optimization method for production and financial investment planning problem, in which the exchange rate is uncertain and characterized by possibility distribution. Ammar and Khalifa [9] developed a rough interval quadratic programming approach for portfolio selection problem to determine the total variability in the future payments.

There has been relatively little empirical analysis of agency problems at severing funds. Largely due to data restrictions, recent papers by Gompers and Metrick [10] and Kahraman et al. [3] have highlighted the heterogeneity of investment strategies, and ultimately returns, across different types of institutional investors. Bernstein et al. [11] reviewed several of the central issues that face sovereign wealth funds. Sirbiladze et al. [12] introduced a new methodology of making a decision on an optimal investment in several projects.

In this paper, we deal with investment problem with an inexact rough interval number, and then solve the optimization problem for a typical investment problem. The process of optimization is illustrated by a numerical example.

The paper is organized as follows: In section 2, some preliminaries are introduced. In section 3, problem statement is introduced. In section 4, a numerical example is given to illustrate the process of optimization. Finally, some concluding remarks are reported in section 5.

2 Preliminaries

In this section, some of the fundamental definitions and concepts of interval of confidence introduced by Kaufmann and Gupta [13], and rough interval initiated by Lu et al. [14] are reviewed.

Moreover, we recall the following symbols:

$x \wedge y = \text{min}\,(x, y)$;

$x \vee y = \text{max}\,(x, y)$;

and, $x + y = \text{sum}\,(x, y)$.

Definition 1. Let *x* denote a closed and bounded set of real numbers. A rough interval x^R is defined as an interval with known lower and upper bounds but unknown distribution information for *x*:

$$x^R = [x^{(UAI)}, x^{(LAI)}],$$

where $x^{(UAI)}$ and $x^{(LAI)}$ are upper and lower approximation intervals of x^R, respectively.

Definition 2. For a rough interval x^R, we have:

$x^R \geq 0$ if and only if $x^{(UAI)} \geq 0$, and $x^{(LAI)} \geq 0$.

$x^R \leq 0$ if and only if $x^{(UAI)} \geq 0$ and $x^{(LAI)} \leq 0$.

Definition 3. For rough intervals $x^R = [[x^{-(UAI)}, x^{+(UAI)}]:[x^{-(LAI)}, x^{+(LAI)}]]$, and $y^R = [[y^{-(UAI)}, y^{+(UAI)}]:[y^{-(LAI)}, y^{+(LAI)}]]$, when $x^R \geq 0$ and $y^R \geq 0$, we have:

(i) $x^R(+)y^R = [[x^{-(UAI)} + y^{-(UAI)}, x^{+(UAI)} + y^{+(UAI)}]:[x^{-(UAI)} + y^{-(UAI)}, x^{+(LAI)} + y^{+(LAI)}]]$

(ii) The order relation $"\leq"$ is defined by:

$x^R \leq y^R$ if and only if $x^{-(UAI)} \leq y^{-(UAI)}$, $x^{+(UAI)} \leq y^{+(UAI)}$, $x^{-(LAI)} \leq y^{-(LAI)}$, and $x^{+(LAI)} \leq y^{+(LAI)}$

(iii) $[[x^{-(UAI)}, x^{+(UAI)}]:[x^{-(LAI)}, x^{+(LAI)}]](\vee)[[y^{-(UAI)}, y^{+(UAI)}]:[y^{-(LAI)}, y^{+(LAI)}]]$
$= [[x^{-(UAI)} \vee y^{-(UAI)}, x^{+(UAI)} \vee x^{+(UAI)}]:[x^{-(LAI)} \vee y^{-(LAI)}, x^{+((LAI)} \vee y^{+(LAI)}]]$

3 Problem Statement

Consider that investor has at his disposal \$ *N* millions for investment in *L* possible production programs I, II, …, *L*. The expected profit for a period *p* are not known, but they will be estimated and given in the form of an inexact rough interval numbers.

Our objective is to allocate the investment in the available L assets in seek a way to maximize the total expected return, for a fixed level of risk. Naturally, the investor cannot exceed his/her available wealth \$ *N* millions.

3.1 Notation

We now define:

$f_1(x)$: The profit function for investing in I,
$f_2(x)$: The profit function for investing in II,
$\vdots$
$f_n(x)$: The profit function for investing in *L*,
$F_{1,2}(\ell)$: The optimal profit, where ℓ is invested in I and II together,
$F_{1,2,3}(\ell)$: The optimal profit, where ℓ is invested in I, II, and III together,
$\vdots$
$F_{1,2,3,\ldots,n}(\ell)$: The optimal profit, where ℓ is invested in I, II, III and *L* together.

4 Numerical Example

Consider an investor has at his disposal \$ 10 millions for investment in four possible production programs, I, II, III, and IV. The expected profits for a three years are not known, but estimated and given in an exact rough intervals as shown in Table 1.

Firstly, compute $F_{1,2}(\ell = 2)$

(a) $f_1(0)(+)f_1(2) = 0(+)[[0.20, 0.26]:[0.21, 0.252]]$
$= [[0.20, 0.26]:[0.21, 0.25]]$

(b) $f\ (1)(+)f_2(1) = [[0.25, 0.30]:[0.26, 0.29]](+)[[0.20, 0.26]:[0.21, 0.25]]$
$= [[0.45, 0.56]:[0.47, 0.54]].$

(c) $f\ (2)(+)f_2(0) = [[0.25, 0.30]:[0.26, 0.29]](+)0$
$= [[0.25, 0.30]:[0.26, 0.29]].$

Comparing the intervals in (a), (b) and (c), we find that the optimal profit or the optimal policy is obtained by investing \$ 1 million I, and \$1 million in II with the total profit (or optimum policy) being $\$[[0.45, 0.56]:[0.47, 0.54]]$ millions.

We will now present the computation on of the optimal profits in investments in I and II for various values of ℓ as:

$$F_{1,2}(\ell) = \operatorname*{Max}_{x+y=\ell} (f_1(x)(+)f_2(y)), \tag{1}$$

Maximum return can be computed using the criterion illustrated in definition 4. the results of these computations are given in Table 2.

Let us now compute $F_{1,2,3}(\ell)$, the optimal return on the investments in I, II and III for various values of ℓ as:

$$F_{1,2,3}(\ell) = \operatorname*{Max}_{x+y=\ell} (f_{1,2}(x)(+)f_3(y)), \tag{2}$$

The results of these computations are given in Table 3.

Now, let us compute $F_{1,2,3,4}(\ell)$, the optimal return on the investments in I, II, III and IV for various values of ℓ as:

$$F_{1,2,3,4}(\ell) = \operatorname*{Max}_{x+y=\ell} (f_{1,2,3}(x)(+)f_4(y)), \tag{3}$$

The results of these computations are given in Table 4.

Table 1. Return on an investment for a period of three years

Investment	Profit investing in I	Profit investing in II	Profit investing in III	Profit investing in IV
0	0	0	0	0
1	[[0.25, 0.30] : [0.26, 0.29]]	[[0.20, 0.26] : [0.21, 0.25]]	[[0.12, 0.16] : [0.13, 0.14]]	[[0.19, 0.24] : [0.20, 0.22]]
2	[[0.40, 0.48] : [0.41, 0.45]]	[[0.33, 0.43] : [0.35, 0.40]]	[[0.21, 0.26] : [0.22, 0.25]]	[[0.35, 0.42] : [0.36, 0.39]]
3	[[0.58, 0.71] : [0.59, 0.65]]	[[0.48, 0.60] : [0.50, 0.56]]	[[0.43, 0.52] : [0.45, 0.47]]	[[0.35, 0.48] : [0.36, 0.46]]
4	[[0.70, 0.85] : [0.71, 0.80]]	[[0.50, 0.67] : [0.55, 0.60]]	[[0.45, 0.51] : [0.46, 0.50]]	[[0.40, 0.52] : [0.42, 0.50]]
5	[[0.81, 1.01] : [0.83, 0.85]]	[[0.60, 0.76] : [0.96, 0.75]]	[[0.53, 0.66] : [0.54, 0.65]]	[[0.51, 0.58] : [0.52, 0.53]]
6	[[0.95, 1.11] : [0.97, 1.05]]	[[0.70, 0.90] : [0.72, 0.85]]	[[0.70, 0.74] : [0.71, 0.73]]	[[0.55, 0.58] : [0.56, 0.57]]
7	[[0.95, 1.16] : [1.06, 1.11]]	[[0.83, 0.90] : [0.84, 0.87]]	[[0.76, 0.83] : [0.77, 0.81]]	[[0.56, 0.59] : [0.57, 0.58]]
8	[[1.10, 1.30] : [1.22, 1.27]]	[[0.85, 0.90] : [0.86, 0.89]]	[[0.89, 0.95] : [0.92, 0.94]]	[[0.58, 0.61] : [0.59, 0.60]]
9	[[1.24, 1.42] : [1.30, 1.35]]	[[0.88, 0.93] : [0.89, 0.91]]	[[0.95, 1.02] : [0.96, 1.00]]	[[0.58, 0.61] : [0.59, 0.60]]
10	[[1.35, 1.50] : [1.39, 1.47]]	[[0.90, 0.94] : [0.91, 0.93]]	[[0.98, 1.08] : [1.00, 1.05]]	[[0.59, 0.64] : [0.60, 0.63]]

Table 2. Optimal policy using rough interval with investments in I and II

ℓ	$f_1(x)$	$f_2(x)$	$F_{1,2}(\ell)$	Optimal policy with I and II
0	0	0	0	(0, 0)
1	[[0.25, 0.30] : [0.26, 0.29]]	[[0.20, 0.26] : [0.21, 0.25]]	[[0.25, 0.30] : [0.26, 0.29]]	(1, 0)
2	[[0.40, 0.48] : [0.41, 0.45]]	[[0.33, 0.43] : [0.35, 0.40]]	[[0.45, 0.56] : [0.47, 0.54]]	(1, 1)
3	[[0.58, 0.71] : [0.59, 0.65]]	[[0.48, 0.60] : [0.50, 0.56]]	[[0.60, 0.74] : [0.62, 0.70]]	(2, 1)
4	[[0.70, 0.85] : [0.71, 0.80]]	[[0.50, 0.67] : [0.55, 0.60]]	[[0.78, 0.97] : [0.80, 0.90]]	(3, 1)
5	[[0.81, 1.01] : [0.83, 0.85]]	[[0.60, 0.76] : [0.69, 0.75]]	[[0.91, 1.10] : [0.94, 1.05]]	(3, 2)
6	[[0.95, 1.11] : [0.97, 1.05]]	[[0.70, 0.90] : [0.72, 0.85]]	[[1.06, 1.31] : [1.09, 1.21]]	(3, 3)
7	[[0.95, 1.16] : [1.06, 1.11]]	[[0.83, 0.90] : [0.84, 0.87]]	[[1.18, 1.45] : [1.21, 1.36]]	(4, 3)
8	[[1.10, 1.30] : [1.22, 1.27]]	[[0.85, 0.90] : [0.86, 0.89]]	[[1.29, 1.61] : [1.33, 1.41]]	(5, 3)
9	[[1.24, 1.42] : [1.30, 1.35]]	[[0.88, 0.93] : [0.89, 0.91]]	[[1.43, 1.71] : [1.47, 1.61]]	(6, 3)
10	[[1.35, 1.50] : [1.37, 1.47]]	[[0.90, 0.94] : [0.91, 0.93]]	[[1.45, 1.78] : [1.47, 1.65]]	(6, 4)

Table 3. Optimal policy using rough interval with investments in I, II and III

ℓ	$F_{1,2}(\ell)$	$f_3(x)$	$F_{1,2,3}(\ell)$	Optimal policy with I ,II and III
0	0	0	0	(0, 0, 0)
1	[[0.25, 0.30] : [0.26, 0.29]]	[[0.12, 0.16] : [0.13, 0.14]]	[[0.25, 0.30] : [0.26, 0.29]]	(1, 0, 0)
2	[[0.45, 0.56] : [0.47, 0.54]]	[[0.21, 0.26] : [0.22, 0.25]]	[[0.45, 0.56] : [0.47, 0.54]]	(1, 1, 0)
3	[[0.60, 0.74] : [0.62, 0.70]]	[[0.43, 0.52] : [0.45, 0.47]]	[[0.60, 0.74] : [0.62, 0.70]]	(2, 1, 0)
4	[[0.78, 0.97] : [0.80, 0.90]]	[[0.45, 0.51] : [0.46, 0.50]]	[[0.78, 0.97] : [0.80, 0.90]]	(3, 1, 0)
5	[[0.91, 1.10] : [0.94, 1.05]]	[[0.53, 0.66] : [0.54, 0.65]]	[[0.91, 1.10] : [0.94, 1.05]]	(3, 2, 0)
6	[[1.06, 1.31] : [1.09, 1.21]]	[[0.70, 0.74] : [0.71, 0.73]]	[[1.03, 1.26] : [1.07, 1.17]]	(2, 1, 3)
7	[[1.18, 1.45] : [1.21, 1.36]]	[[0.76, 0.83] : [0.77, 0.81]]	[[1.21, 1.49] : [1.25, 1.37]]	(3, 1, 3)
8	[[1.29, 1.61] : [1.33, 1.41]]	[[0.89, 0.95] : [0.90, 0.94]]	[[1.34, 1.66] : [1.39, 1.52]]	(3, 2, 3)
9	[[1.43, 1.71] : [1.47, 1.61]]	[[0.95, 1.02] : [0.96, 1.00]]	[[1.49, 1.83] : [1.54, 1.68]]	(3, 3, 3)
10	[[1.45, 1.78] : [1.47, 1.65]]	[[0.98, 1.08] : [1.00, 1.05]]	[[1.61, 1.97] : [1.66, 1.83]]	(4, 3, 3)

Table 4. Optimal policy using rough interval with investments in I, II, III and IV

ℓ	$F_{1,2,3}(\ell)$	$f_4(x)$	$F_{1,2,3,4}(\ell)$	Optimal policy with I, II, III and IV
0	0	0	0	(0, 0, 0, 0)
1	[[0.25, 0.30] : [0.26, 0.29]]	[[0.19, 0.24] : [0.20, 0.22]]	[[0.25, 0.30] : [0.26, 0.29]]	(1, 0, 0, 0)
2	[[0.45, 0.56] : [0.47, 0.54]]	[[0.35, 0.42] : [0.36, 0.39]]	[[0.45, 0.56] : [0.47, 0.54]]	(1, 1, 0, 0)
3	[[0.60, 0.74] : [0.62, 0.70]]	[[0.35, 0.48] : [0.36, 0.46]]	[[0.64, 0.80] : [0.67, 0.76]]	(1, 1, 0, 1)
4	[[0.78, 0.97] : [0.80, 0.90]]	[[0.40, 0.52] : [0.42, 0.50]]	[[0.79, 0.98] : [0.82, 0.92]]	(2, 1, 0, 1)
5	[[0.91, 1.10] : [0.94, 1.05]]	[[0.51, 0.54] : [0.52, 0.53]]	[[0.97, 1.21] : [1.00, 1.12]]	(3, 1, 0, 1)
6	[[1.03, 1.26] : [1.07, 1.17]]	[[0.55, 0.58] : [0.56, 0.57]]	[[1.13, 1.39] : [1.16, 1.29]]	(3, 1, 0, 2)
7	[[1.21, 1.49] : [1.25, 1.37]]	[[0.56, 0.59] : [0.57, 0.58]]	[[1.26, 1.56] : [1.30, 1.44]]	(3, 2, 0, 2)
8	[[1.34, 1.66] : [1.39, 1.52]]	[[0.58, 0.61] : [0.59, 0.60]]	[[1.40, 1.73] : [1.45, 1.59]]	(3, 1, 3, 1)
9	[[1.49, 1.83] : [1.54, 1.68]]	[[0.58, 0.61] : [0.59, 0.60]]	[[1.56, 1.91] : [1.61, 1.76]]	(3, 1, 3, 2)
10	[[1.61, 1.97] : [1.66, 1.83]]	[[0.59, 0.64] : [0.60, 0.63]]	[[1.69, 2.08] : [1.75, 1.91]]	(3, 2, 3, 2)

Hence, the best investment for \$ 10 millions as computed in Table 4. \$ 3 millions in I with an inexact rough interval optimal return

\$ [[0.85, 0.71] : [0.59, 0.65]] millions,

\$ 2 millions in II with an inexact rough interval optimal return

\$ [[0.33, 0.34] : [0.35, 0.40]] millions,

\$ 3 millions in III with an inexact rough interval optimal return

\$ [[0.43, 0.52] : [0.45, 0.47]] millions,

\$ 2 millions in IV with an inexact rough interval optimal return

\$ [[0.35, 0.42] : [0.36, 0.39],

Thus the total optimal return with an inexact rough intervals on a \$ 10 millions investment is

\$ [[1.69, 2.08] : [1.75, 1.91]] millions.

5 Concluding Remarks

In this paper, investment problem with an inexact rough intervals has been introduced. A dynamic programming approach has been applied to obtain an inexact optimal rough interval return. The significant benefit of using such approach than the others where the decision maker facing a problem including ambiguity in the data of the problem. From this study, it has been cleared that the investment methodology has been provided the framework in which the planned investment is fully investigated and all options explored so to ensure that it is aligned with the organizations business objectives and strategies direction. The process of optimization has been illustrated by a numerical example.

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Competing Interests

Authors have declared that no competing interests exist.

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