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Performance Evaluation of Unreliable M(t)/M(t)/n/n Queueing System

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Authors' contributions

This work was carried out in equal collaboration between all authors. Author SG designed the study, and wrote the first draft of the manuscript. Author RPG performed the statistical analysis and numerical calculations. Author GBT managed literature searches and wrote the protocol. All authors read and approved the final manuscript.

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ABSTRACT

This paper deals with the performance evaluation of multi-server queuing system subject to breakdowns under transient frame work. The system does not accept the queue of the waiting customers. If the new customer upon its arrival finds 'n' customers already present in the system, then it is rejected. Customers arrive to the system in Poisson fashion and are served exponentially. The main purpose of this paper is to evaluate some performance measures- the proportion of lost customers, mean number of customers in service, utilization factor of servers, mean number of broken servers and utilization of repair capacity, all at any instant. By varying different parameters, the system behavior is examined with the help of numerical illustrations by using computing software so as to show the model under study has ample practical applications.

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1. INTRODUCTION

It is a multi server queueing model where customers arrive in a Poisson process and service time is distributed exponentially. The system is arranged to serve for 'n' customers. That is, if there are more than 'n' customers, they cannot get the service. This paper deals with transient analysis of M(t)/M(t)/n/n queueing model with time dependent arrival and service rates wherein we set up the system of transient balance differential –difference equations and solve them for the probability distribution for any time 't' and show the effect of arrival rate, service rate and time over the probability distribution.

In several research works in queueing theory, the study of queueing model has been made under the non-failure servers but this is not realistic. In many real life situations, servers remain to be the technical devices such as RAM in computer, camera in unman aerial vision (UAV), sensor in automatic operating machines where devices are subject to break down and such a device is termed as unreliable. In other words, non-reliable server means that the server is typically subject to unpredictable breakdowns. Transient study of multi-server queueing system under the provision of unreliable concept has been found very rare in the literature.

The remaining part of the paper is planned as follows: Section II describes the brief literature review where previous research works are explained briefly. Section III presents mathematical model and analysis regarding the balance equations in the form of differential-difference equations. These equations are sketched in Fig. 2 as well. Section IV presents the numerical results and interpretations where the real life situation of this model is graphically represented with the help of MATLAB programming. Section V describes few of the applications of this model. Finally, section VI concludes the paper.

2. BRIEF LITERATURE REVIEW

Recently Sridhar and Pitchai [1] studied two sever queueing system with heterogeneous batch service and the expression for the expected queue length by using the generating function approach has been derived. Suhasini et al. [2] derived the explicit formulas for average number of customers in each queue, the probability of emptiness of the system, the average waiting time in the queue and in the system. Throughput of the nodes and the variance of the number of customers in the queue have also been calculated. Shekhar et al. [3] dealt with a model of sensitivity analysis with respect to the uniformly distributed batch size arrivals. Multi-component machining system with the provision that, when the machine breakdowns, servers repairs it but servers themselves fail unpredictably, is studied to obtain some of the performance measures that makes the system more practicable. Yang et al. [4] studied a machine repair problem in which servers are repairmen and undergo breakdown (unreliable) with multiple vacations. Their citations for the various performance indices are also notable. Ayyappan and Devipriya [5] studied the batch service system with single server with exponential service time distribution and Poisson arrival rate under the assumption of Catastrophe. Various measures of performance such as analytical solution for mean number of customers and variance of the system, closed form solution for probability of number of customers in the queue during the idle and busy server period has also been obtained. Indra and Renu [6] presented two-dimensional M/M/1 queue under the vacations provision and obtained the repair times, servers' service times, vacation times, and breakdown times. Zhang et al. [7] studied various performance indices on M/M/1 queue and obtained various performance measures. Later on Wang et al. [8] further studied the removable and non-reliable server of $M/H_2/2$ queueing model under the two types of hyper-exponential distribution for the service times and investigated the optimal cost analysis. Kalidass and Ramanath [9] obtained the explicit expressions for time dependent probabilities of the M/M/1 queue with server vacations under a waiting server and also made the sensitivity analysis of the model.

Dorda [10] contributed to modeling and simulation of a Markov multi-server queueing system subject to breakdowns and with an ample repair capacity where the system does not form the queue of waiting customers. Ghimire et al. [11] calculated bulk queueing model with the fixed batch size 'b' and has obtained the expressions for mean waiting time in the queue, mean time spent in the system, mean number of customers (work pieces) in the queue. Jain et al. [12] analyzed performance of primary and secondary unreliable servers in a multi-component machining system in which the smooth functioning of the system has a provision of warm spares that may not be perfect in switching of the failed operating unit. In the study of Jain et al. primary server is prone to complete or partial breakdown whereas secondary server faces only complete breakdown independently or simultaneously due to common cause. Jain and Bhargava [13] undertook the research for explanation of the N-policy machine repair queueing system with mixed types of standby system and calculation of the explicit expressions for expected number of standby units, probabilities of servers being busy, idle and broken, system reliability and mean time to system failure. Jain et al. [14] presented the machining queueing system with standby spare parts supports and balking and reneging assumptions. Mohit et al. [15] studied time varying arrival and a model for service queue and obtained the probabilities for expected length of system by the generating function approach. They used the Runge-Kutta approximation technique to solve the system of differential-difference equations. Kishan and Jain [16] used the Monte-Carlo simulation analysis to study the repairable machine repair system with two non-identical unit standby systems and obtained the mean time to the system failure.

Al-Seedy et al. [17] analyzed M/M/C queue with balking and reneging by using the generating function technique under the time dependent assumptions and presented the real life situations of the model in inventory system. Gupta [18] made the study of some Poissonian queuesing systems and non- Poissonian queueing system with comparative study between them and their applications in the airport delay estimations. Chandrasekaran and Saravanarajan [19] calculated transient analysis of M/M/1 queue by using the continued fractions method for the solution for system size probabilities subject to the catastrophes, server failures and repairs. Grassmann [20] formulated for the analytic study for the busy period, reliability and availability with transient solution of M/M/1 queueing model giving the sparse transition matrix. Ghimire and Ghimire [21] dealt M/M/1 queue with heterogeneous arrival and departure with the provision of server vacations and breakdowns. They also calculated the mean queue length, mean waiting time in the queue and in the system, average number of customers in the system. Yang and Wu [22] considered the study on M/M/1/N queue with working breakdowns and server vacations where arrivals occur according to a Poisson process. The server is subject to breakdown and repairs while in operation. At failure times, the server still works at a lower service rate rather than completely stopping service. A numerical technique based on the fourth-order Runge–Kutta method is used to compute the transient state probabilities. Ashour and Jha [23] studied numerical transient-state solutions of queueing systems and observed the feasibility of applying numerical techniques for obtaining transient solutions. Takács [24] has formulated the time dependence of Palm's loss formula applicable in the telecommunication. Isguder and Uzunoglu-Kocer [25] dealt with a multi-server, finite-capacity queuing system with recurrent input and no waiting line where the servers are heterogeneous and independent of each other. Also, when all servers are busy at a time of an arrival, that arrival must leave the system without being served.

3. MATHEMATICAL MODEL AND ANALYSIS

There are some important assumptions and notations needed for this model and we have the following assumptions for our model:

(i) Multi-server queuing systems formed by 'n' homogeneous parallel servers are subjected to failure-free and broken downs independently.
(ii) Incoming customer which does not find an idle server is rejected, i.e. no queue is formed.
(iii) Customers come to the system according to the Poisson process with the rate λ and are served with exponential service time distribution rateμ.
(iv) The time of failure-free state is an exponential random variable with the parameter η so that the mean time of failure-free state is $\frac{1}{\eta}$.
(v) Server repair time has an exponential distribution with parameter ν that means $\frac{1}{\nu}$ is the mean time that a server takes to repair the machine.
(vi) The pair (i, j) where $i = 0,1,2,\cdots,n$ are the number of broken servers and $j = 0,1,2,\cdots,n-i$ are the number of the customers in the service or in the system.
(vii) There is a provision that a pre-empted customer is returned to the pool of waiting

customers and is served by the first available server. Under this provision, if any of the servers go for breakdown, that server will resume the service to the customer after the repair. Server will start the service for the interrupted customer right from the beginning.

For these different notations and assumptions, transition probability distributions for the different states have been described in Fig. 1. Since the service and the arrival depend on time, differentiation of every state with respect to time is equal to the difference of sum of the incoming state to that state and the sum of the outgoing state from that state. At the top most row of the figure none of the servers are broken. So, all the first numbers are zero. In the second row, one of the servers is broken therefore only (n-1) customers can get the service which is represented by the last number in the second row. Similarly, the number of broken servers is shown by the first number of each row. There are total of 'n' customers in the system to get the service and hence, if all the servers are broken down no service will be provided. Therefore, in the last row all the servers are subject to breakdown indicating that none of the customers can get the service.

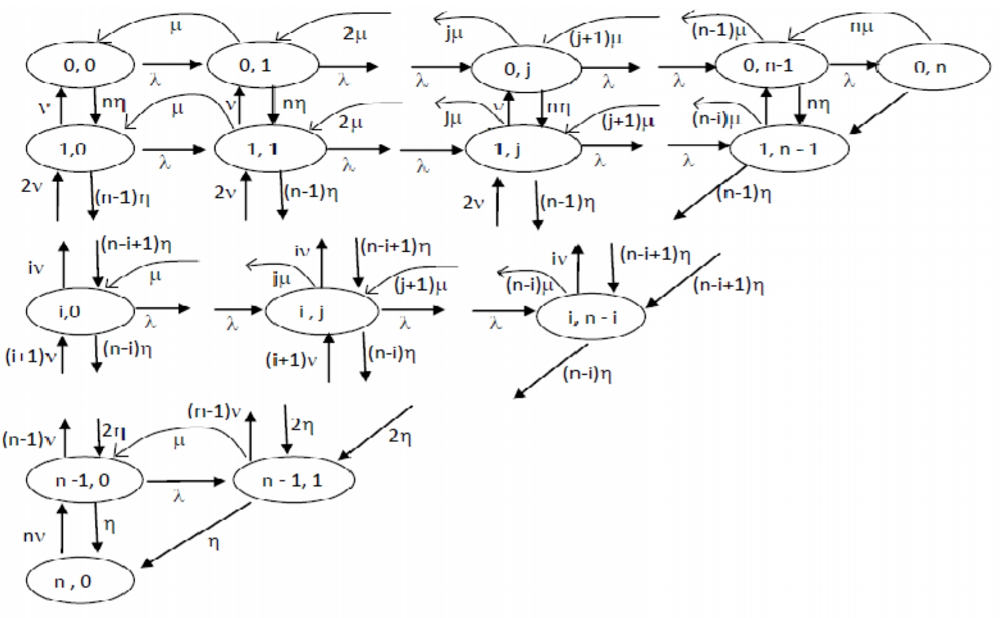

Fig. 1. Transient state transition diagram

Source: Michal Dorda [10]

For our model the differential difference equations from Fig. 1 are:

$$\frac{d}{dt}P_{(0,0)} = -[\lambda(t) + n\eta]P_{(0,0)}(t) + \mu(t)P_{(0,1)}(t) + \nu P_{(1,0)}(t) \quad (1)$$

$$\frac{d}{dt}P_{(0,j)}(t) = -[\lambda(t) + j\mu(t) + n\eta]P_{(0,j)}(t) + \lambda(t)\,P_{(0,j-1)}(t) + (j+1)\mu(t)P_{(0,j+1)}(t) + \nu P_{(1,j)}(t),$$
$$j = 1,2,\ldots,n-1 \quad (2)$$

$$\frac{d}{dt}P_{(0,n)}(t) = -[n\mu(t) + n\eta]P_{(0,n)}(t) + \lambda(t)\,P_{(0,n-1)}(t) \quad (3)$$

$$\frac{d}{dt}P_{(i,0)}(t) = -[\lambda(t) + (n-i)\eta + i\nu]P_{(i,0)}(t) + \mu(t)P_{(i,1)}(t) + (n-i+1)\eta P_{(i-1,0)}(t) + (i+1)\nu P_{(1+1,0)},$$
$$i = 1,2,\ldots,n-1 \quad (4)$$

$$\frac{d}{dt}P_{(i,j)}(t) = -[\lambda(t) + j\mu(t) + (n-i)\eta + i\nu]P_{(i,j)}(t) + \lambda(t)\,P_{(i,j-1)}(t) + (j+1)\mu(t)P_{(i,j+1)}(t) +$$
$$(n-i+1)\eta P_{(i-1,j)}(t) + (i+j)\nu P_{(i+1,j)}(t),\ i = 1,2,\ldots,n-2,\ j = 1,2,\ldots,n-i-1 \quad (5)$$

$$\frac{d}{dt}P_{(i,n-i)}(t) = -[(n-i)\mu(t) + (n-i)\eta + i\nu]P_{(i,n-i)}(t) + \lambda(t)P_{(i,n-i+1)}(t) + (n-i+1)\eta P_{(i-1,n-i)}(t) + (n-i+1)\eta P_{(i-1,n-i+1)}(t),\ i = 1,2,\ldots,n-1 \quad (6)$$

$$\frac{d}{dt}P_{(n,0)}(t) = -n\nu P_{(n,0)}(t) + \eta P_{(n-1,0)}(t) + \eta P_{(n-1,1)}(t) \quad (7)$$

The co-efficient matrix of the above system of equations is denoted by Q and hence it is derived as follows:

$$Q = \begin{pmatrix} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \\ Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} \\ Q_{11} & Q_{12} & Q_{13} & Q_{14} & Q_{15} \\ Q_{16} & Q_{17} & Q_{18} & Q_{19} & Q_{20} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} & Q_{25} \end{pmatrix} \quad (8)$$

where the matrices Q_1, Q_2, ...,Q_{25}, are the sub-matrices which can be expressed distinctly as below:

$$Q_1=\begin{pmatrix} -[\lambda+n\eta] & \mu & 0 & \ldots & 0 \\ \lambda & -[\lambda+\mu+n\eta] & 2\mu & \ldots & 0 \\ 0 & \lambda & -[\lambda+2\mu+n\eta] & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -[\lambda+(n-2)\mu+n\eta] \end{pmatrix};\ Q_2=\begin{pmatrix} 0 & 0 & \nu & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (n-1)\mu & 0 & 0 & 0 & \ldots \end{pmatrix};$$

$$Q_3=\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots \end{pmatrix},\ Q_4=\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix},\ Q_5=\begin{pmatrix} 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 0 & 0 \end{pmatrix},\ Q_6=\begin{pmatrix} 0 & 0 & 0 & \ldots & \lambda \\ 0 & 0 & 0 & \ldots & 0 \\ n\eta & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & 0 \end{pmatrix},$$

$$Q_7=\begin{pmatrix} -[\lambda+(n-1)\mu+n\eta] & n\mu & 0 & 0 & \ldots \\ \lambda & -(n\mu+n\eta) & 0 & 0 & \ldots \\ 0 & 0 & -[\lambda+(n-1)\eta+\nu] & 2\nu & \ldots \\ 0 & 0 & (n-1)\eta & -[\lambda+(n-2)\eta+2\nu] & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},$$

$$Q_8=\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},\ Q_9=\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},\ Q_{10}=\begin{pmatrix} 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},$$

$$Q_{11}=\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & n\eta & 0 & \ldots & 0 \\ 0 & 0 & n\eta & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & 0 \end{pmatrix},\ Q_{12}=\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & \lambda & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},$$

$$Q_{13}=\begin{pmatrix} -[\lambda+\eta+(n-1)\nu] & 0 & 0 & 0 & \ldots \\ 0 & -[\lambda+\mu+(n-1)\eta+\nu] & 2\mu & 0 & \ldots \\ 0 & 0 & -[\lambda+2\mu+(n-1)\eta+\nu] & 3\mu & \ldots \\ 0 & 0 & \lambda & -[\lambda+3\mu+(n-1)\eta+\nu] & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},$$

$$Q_{14}=\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 2\nu & 0 & 0 & \ldots & 0 \\ 0 & 3\nu & 0 & \ldots & 0 \\ 0 & 0 & 4\nu & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},\ Q_{15}=\begin{pmatrix} 0 & \ldots & \mu & n\nu & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix},\quad Q_{16}=\begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix},$$

$$Q_{17} = \begin{pmatrix} 0 & 0 & 0 & \lambda & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ n\eta & n\eta & 0 & 0 & \dots \end{pmatrix}, \quad Q_{18} = \begin{pmatrix} 0 & (n-1)\eta & 0 & 0 & \dots \\ 0 & 0 & (n-1)\eta & 0 & \dots \\ 0 & 0 & 0 & (n-1)\eta & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots \end{pmatrix},$$

$$Q_{19} = \begin{pmatrix} -[\lambda+\mu+(n-2)\eta+2\nu] & 2\mu & 0 & \dots & 0 \\ \lambda & -[\lambda+2\mu+(n-2)\eta+2\nu] & 3\mu & \dots & 0 \\ 0 & \lambda & -[\lambda+3\mu+(n-2)\eta+3\nu] & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -[(n-1)\mu+(n-1)\eta+\nu] \end{pmatrix},$$

$$Q_{20} = \begin{pmatrix} 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & 0 \end{pmatrix}, Q_{21} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}, Q_{22} = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \end{pmatrix},$$

$$Q_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \end{pmatrix}, \quad Q_{24} = \begin{pmatrix} 0 & 0 & 0 & \dots & (n-1)\eta \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix},$$

$$Q_{25} = \begin{pmatrix} -[(n-2)\mu+(n-2)\eta+2\nu] & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & .. & \dots \\ 0 & \dots & -[2\mu+2\eta+(n-2)\nu] & 0 & 0 \\ 0 & \dots & 2\eta & -[\mu+\eta+(n-1)\nu] & 0 \\ 0 & \dots & 0 & \eta & -n\nu \end{pmatrix}$$

Denoting P(t) $=[P_{0,0}(t),...,P_{n,n}(t)]^T$; $P(0) = [1,0,\dots,0]^T$, we obtain the system of equations as

$$\frac{d}{dt} P_{i,j}(t) = Q.\, P_{i,j}(t) \qquad (9)$$

Some other performance measures are also observed in this study which is described as follows:

(i) Transition probability distribution over initial times for various states.
(ii) If $\rho(t)$ is the traffic intensity, probability that there are $s < n$ customers in the system in any time Takács (1961).

$$t = \frac{\frac{\rho(t)^s}{s!}}{\sum_{i=0}^{n} \frac{\rho(t)^i}{i!}}$$

(iii) Time-dependent mean number of customers in the system $L_s = \rho(t)(1 - B_p(t))$.

$$\text{Where } B_p(t) = \frac{\frac{\rho(t)^n}{n!}}{\sum_{i=0}^{n} \frac{\rho(t)^i}{i!}}$$

4. NUMERICAL RESULTS AND INTERPRETATIONS

Numerical results have been obtained by using the Euler numerical approximation method. For the numerical computations, the values of the variables taken are as follows: h = 0.001, λ = 0.2, μ = 0.3, η = 0.1, ν = 0.2, n = 5, P(0) = [1,0,0,0,0,0,0,0,0,0,0,...,0,0,0,0,0,0,0]. Based on these values of the variables MATLAB programming is used to plot the graphs.

In Fig. 2 we can see that, as the arrival rate increases the probability distribution decreases gradually. It is realistic also because server will be busier. Similarly, Fig. 3 displays that as the service rate increases the initial probability distributions decrease with the restriction on time that shows the system we have studied is very close the real life situations. Likewise, in Fig. 4 initial probability distribution over time period is not equally distributed. As, the time passes probability distribution decreases gradually.

Fig. 5 explains that when the time goes on, the customers to the system increases and the probability of the number of customers which will not exceed the fixed number will also increase. That is obviously we have experienced in real life also. Likewise, Fig. 6 depicts that, as the time passes on, the number of customers will also be higher with the higher arrival rates.

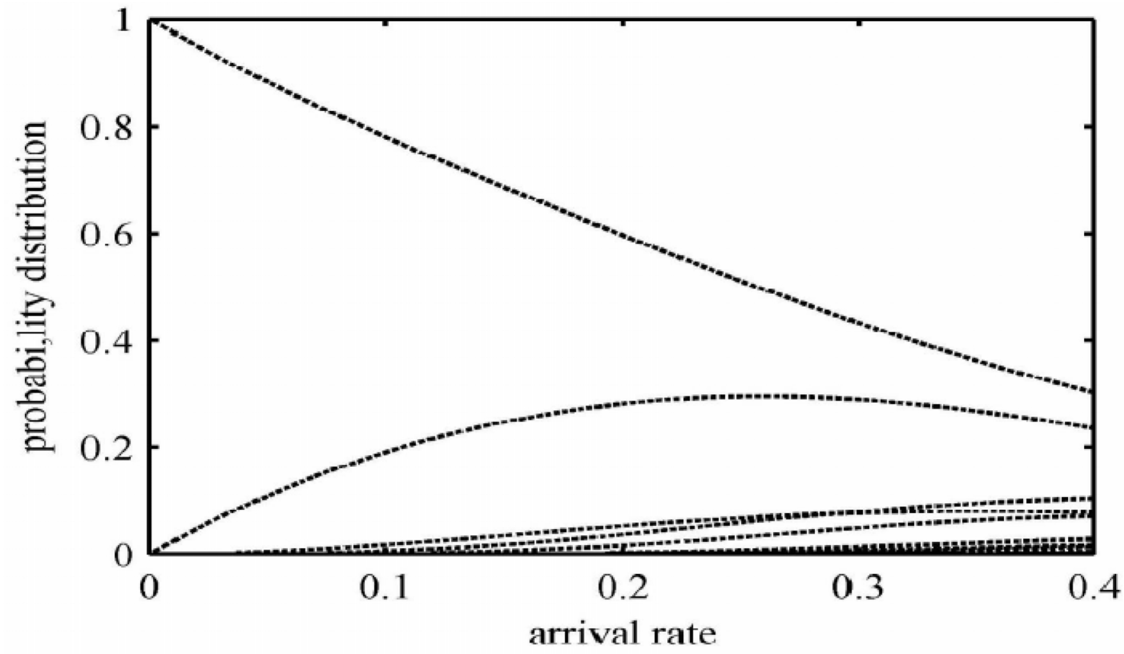

Fig. 2. Probability distribution vs arrival rate

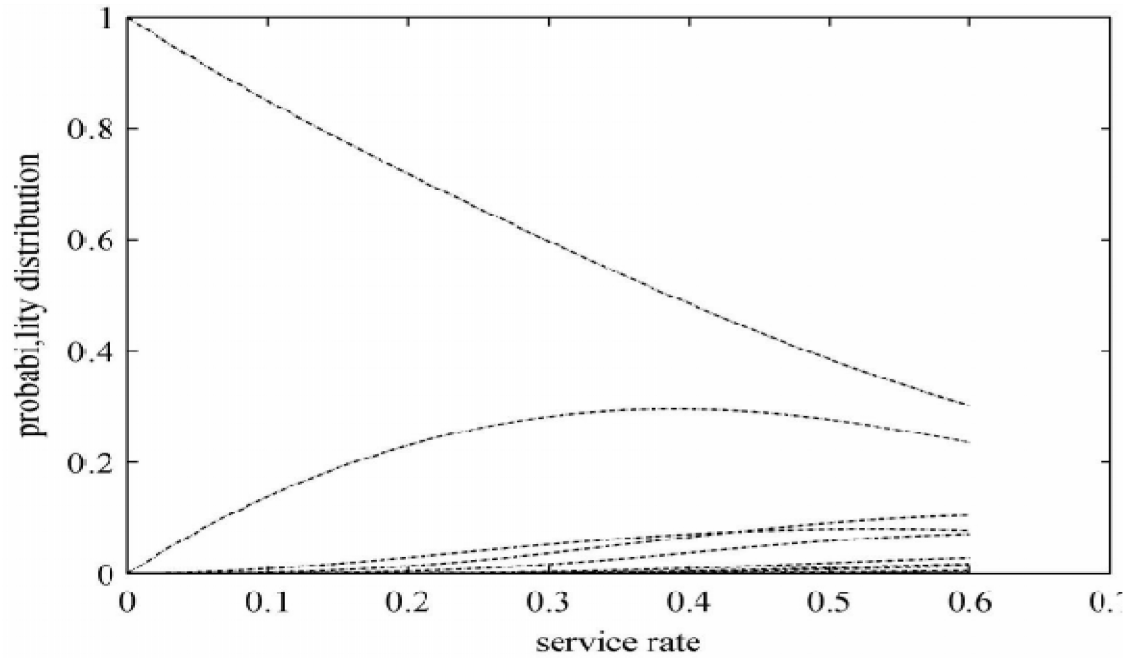

Fig. 3. Probability distribution vs service rate

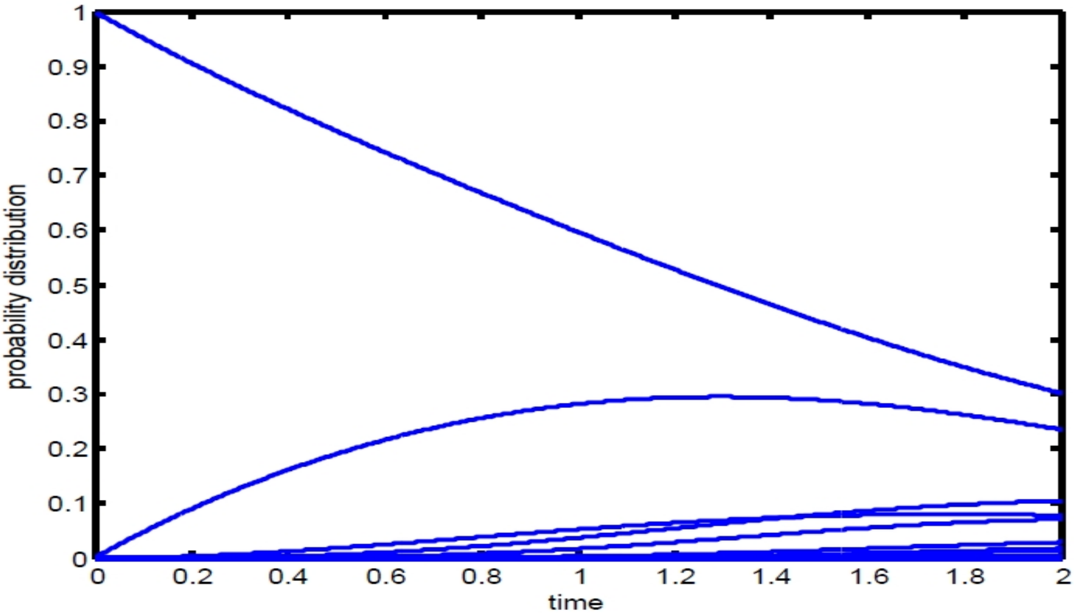

Fig. 4. Probability distribution vs time

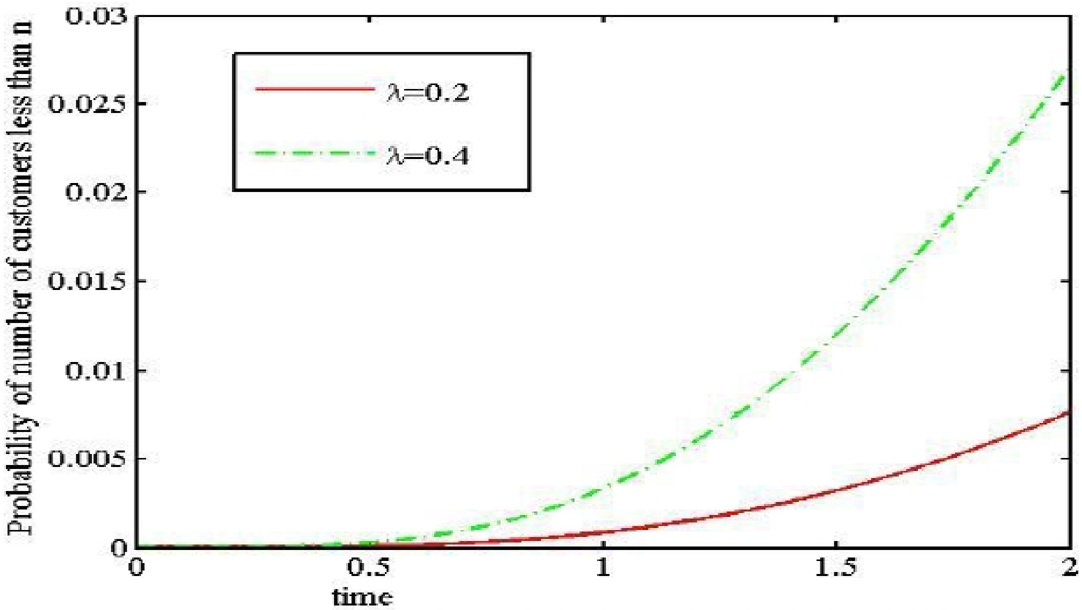

Fig. 5. Probability of less than 'n' customers vs time

5. APPLICATIONS

The queueing system has a wide area of application in the real world. We describe some of the applications of our study in the followings.

We can experience the application of this model in the internet queueing system. Considering every port in a router as a server and every computer connected to that router as a customer. One port in the router is supposed to serve only one computer. If a particular port does not work, there will be one less computer to get the service. In the banking queueing system, only one customer is served at a time. If a particular server fails to serve, the upcoming customer will go for the idle server indicating that total number of customers will be reduced by 1 to get the service. When the broken server gets repaired that will again start serving the customers.

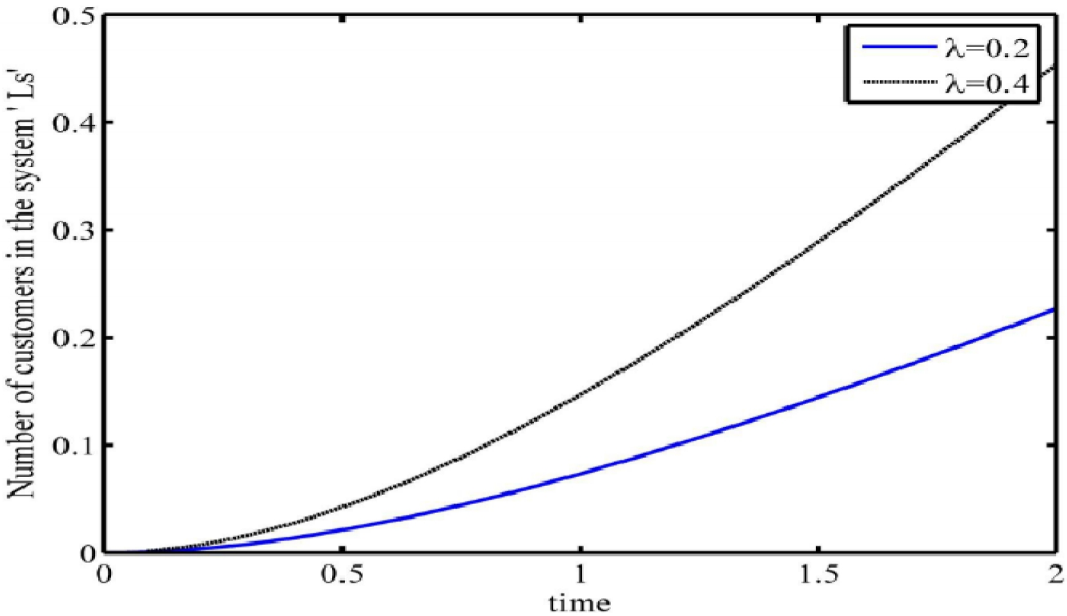

Fig. 6. Number of customers in the system vs time

We can experience these types of applications in many other situations also. We have noticed many counseling programs organized by different consultancies. Counselors expect to council individually. The same situation arises when we go to visit the doctor. A doctor can observe only one patient at a time and the waiting patient cannot get the service at the same time. The production company is also the next area where we can observe its application. Every item produced will first be finished and then only will go the next production. If a device producing the particular item fails to work, it will start the next production only after the device is repaired. Model has been studied for initial probability distributions for very small scale of parameters and our model is closed and more general to the model studied by Ashour and Jha [23].

6. CONCLUSION

We use devices in many realistic situations to provide services to the customer. We have experienced that components of those machines are subject to breakdown such as in unman aerial vision (UAV), sensor in automatic operating machines. Under this condition, service will be affected. This realistic situation under the constraint that whenever server is under breakdown, no service is provided has been studied. Moreover, arrival and the service both depend on time and hence we have studied the time-dependent arrival and service rates that makes the model closure to the realistic. Plenty of situations are there in our daily life, where long queues are formed and the customers get the service one after the other, but in this model no long queue will be formed. There are 'n' number of servers to provide the service for 'n' number of customers. If 'j' number of servers is broken down there will be only (n-j) number of customers to get the service.

We can still extend the area of study further, in which, long queue can be formed and customers will wait for the service staying in the queue. If we consider the service for infinite customers that would be more challenging research area.

DISCLAIMER

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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