






# Tearing Modes in Partially Ionized Astrophysical Plasma

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## Abstract

In many astrophysical environments the plasma is only partially ionized, and therefore the interaction of charged and neutral particles may alter both the triggering of reconnection and its subsequent dynamical evolution. We derive the tearing mode maximum growth rate for partially ionized plasmas in the cases of weak and strong coupling between the plasma and the neutrals. In addition, critical scalings for current sheet aspect ratios are presented in terms of Lundquist number and ion–neutral collision frequencies for which the tearing mode becomes fast, or ideal. In the decoupled regime the standard tearing mode is recovered with a small correction that depends on the ion–neutral collision frequency; in the intermediate regime collisions with neutrals are shown to stabilize current sheets, resulting in larger critical aspect ratios for ideal tearing to occur. In the coupled regime, the growth rate depends on the density ratio between ions and neutrals through the collision frequency between these two species.

*Unified Astronomy Thesaurus concepts:* [Solar magnetic reconnection \(1504\)](#); [Plasma astrophysics \(1261\)](#); [Space plasmas \(1544\)](#); [Collision processes \(2065\)](#)

## 1. Introduction

Magnetic reconnection is considered to be an important dynamical mechanism in a variety of astrophysical plasmas (Zweibel & Yamada 2009; Yamada et al. 2010). Without magnetic reconnection, stars and accretion disks would not have coronae, magnetic dynamos would not work, and there would most probably be no supersonic solar wind (e.g., Zweibel & Yamada 2009; Yamada et al. 2010). A complete understanding of magnetic reconnection in astrophysical settings therefore requires explaining how energy accumulates in the magnetic field, how current carrying fields becomes unstable, and how magnetic energy release occurs on short timescales once the reconnection process has been triggered. One of the major difficulties in understanding magnetic reconnection in astrophysical plasmas stems from the fact that classical models of reconnection, starting from the steady-state Sweet–Parker mechanism (Parker 1957; Sweet 1958), or the nonsteady, resistive instabilities (Furth et al. 1963), appeared to be inadequate to explain the observed, transient and explosive release of magnetic energy. More recently, the thin Sweet–Parker current sheets have been shown to be unstable to a fast tearing instability (Biskamp 1986; Shibata & Tanuma 2001; Loureiro et al. 2007). Pucci & Velli (2014, hereafter PV14) showed that, in a resistive framework, current sheet inverse aspect ratios scaling as  $a/L \sim S^{-1/3}$  separate slowly evolving systems from ones that are so unstable they should never form. In the expression,  $a$  is the thickness and  $L$  is the length of the current sheet;  $S = LV_A/\eta_m$ , with  $\eta_m$  the magnetic diffusivity, is the Lundquist number (note that  $S$  may also be written as  $S = \tau_R/\tau_A$  with  $\tau_R$  the resistive diffusion time and  $\tau_A$  the Alfvén crossing time over the scale  $L$ ). They called this regime ideal tearing (IT). While PV14 focused on the problem of the asymptotic limit  $S \rightarrow \infty$ , in which case it is easy to see that the fastest-growing mode of the tearing instability dominates and

determines the inverse aspect ratio scaling, Uzdensky & Loureiro (2016) provided a derivation for finite  $S$  following the full range of unstable modes, ultimately confirming the PV14 result in the large  $S$  limit. Comisso et al. (2016) rely on the effects of initial noise in the development of the instability (which we note might not be correctly represented by tearing eigenfunctions in a simulation transient phase), obtaining a logarithmic correction in  $S$  on the sheet disruption time that again does not modify the PV14 scaling at large  $S$ . Indeed the PV14 scaling was confirmed in numerical simulations by Landi et al. (2015), Tenerani et al. (2015b), Landi et al. (2017), Huang et al. (2017), and extended to recursive reconnection and general plasmoid number scalings by Singh et al. (2019). Subsequently, kinetic effects that play a role once small enough scales are reached were incorporated into the IT scenario as well (Singh et al. 2015; Del Sarto et al. 2016; Pucci et al. 2017).

There are also environments (e.g., solar photosphere and solar chromosphere, solar filaments/prominences, the interstellar medium, dense molecular clouds, protoplanetary disks) where the astrophysical plasmas undergoing reconnection are only partially ionized (see, e.g., Ballester et al. 2018). The ionization degree depends upon the electron–neutral and the electron–ion collision frequencies (Alfvén 1960), while the resulting drag force acting on each species must satisfy momentum conservation for the whole plasma. This means that depending on the ratio between the density of the ions and neutrals (or electrons and neutrals, if their collisions are not negligible), the associated collision frequencies may establish additional characteristic times scales of the system. Depending on the strength of the coupling between ion and neutrals and the dynamical times of magnetic reconnection, the reconnection rate can be affected. There are a number of theoretical studies on tearing mode instability that show the dependence of

the growth rate of the instability on ion–neutral collisions (Zweibel 1989; Zweibel et al. 2011; Singh et al. 2015).

Multifluid MHD simulations show that, as a result of current sheet thinning and elongation, a critical Lundquist number  $S_c$ , is reached in a partially ionized plasma, at which point plasmoid formation starts (Leake et al. 2012, 2013). In such multifluid simulations, during the current sheet thinning, a stage is reached where the neutrals and ions decouple, and a reconnection rate faster than the single-fluid Sweet–Parker prediction is observed. The ion and neutral outflows are well coupled in the multifluid MHD simulations in the sense that the difference between ion and neutral outflow is negligible compared to the magnitude of the ion outflow. Assuming incompressibility and the same pressure gradient for ion and neutrals, in a reduced MHD frame, Zweibel (1989) calculated the growth rate of the classic tearing instability in the so-called *constant-psi* regime (Furth et al. 1963). In this Letter, starting from the model described in Zweibel (1989), we calculate the maximum growth rate for the tearing mode instability in partially ionized plasmas, assuming as a primary source of drag the collisions between ions and neutrals (retaining Coulomb collisions between ions and electrons). We calculate the scaling of the growth rate depending on the coupling, the relative speed of collisions and the growth rate itself. Then, applying the IT criterion, we find, for each regime, the scaling of the critical aspect ratio for which the growth rate depends neither on the Lundquist number nor on the density ratios.

## 2. Tearing Modes in a Partially Ionized Plasma

Consider a one-dimensional current sheet structure in which the magnetic field reverses sign:

$$\mathbf{B}(y) = B(y)\hat{i} = B_0 F\left(\frac{y}{a}\right)\hat{i}, \quad (1)$$

where  $B_0$  is the asymptotic amplitude of the field,  $F$  is an arbitrary odd nondimensional function, whose first derivative provides the current profile. A specific example is given by the Harris current sheet  $F = \tanh(y/a)$ . The dispersion relation for the reconnecting tearing instability depends, in the resistive magnetohydrodynamics (MHD) framework, on the magnetic diffusivity  $\eta$ , the shear-scale  $a$  defining the current sheet thickness, and the wavenumber  $ka$ . As discussed in Del Sarto et al. (2016) and Pucci et al. (2018) for general equilibrium profiles, specifying the function  $F$  results in a different dependence on the wavenumber  $ka$ . This arises from the fact that at a large Lundquist number two regions define the solution structure: a boundary layer of thickness  $2\delta$  around the center ( $y=0$ ) of the current sheet, and outer regions where diffusivity and growth rate may be neglected. Such outer solutions lead to a discontinuity of the first derivative of the perturbing magnetic field at the neutral point (regularized by diffusion in the inner layer): the jump in the gradient of the reconnecting field component is called  $\Delta'$ . Two asymptotic expressions summarize the dispersion relation, depending on whether  $\Delta'\delta/a \ll 1$  (small Delta prime, or  $\Delta'$ , subscript SD), where

$$\gamma_{\text{SD}}\bar{\tau}_A \simeq A^{\frac{4}{5}}\bar{k}^{\frac{2}{5}}(\Delta')^{\frac{4}{5}}\bar{S}^{-\frac{3}{5}} \quad \delta_{\text{SD}}/a \sim (\bar{S}\bar{k})^{-\frac{2}{5}}(\Delta')^{\frac{1}{5}}, \quad (2)$$

where  $A$  is a nondimensional constant, or  $\Delta'\delta/a \gg 1$  (large Delta prime, or  $\Delta'$ , subscript LD),

$$\gamma_{\text{LD}}\bar{\tau}_A \simeq \bar{k}^{\frac{2}{3}}\bar{S}^{-\frac{1}{3}} \quad \delta_{\text{LD}}/a \sim (\bar{S}\bar{k})^{-\frac{1}{3}}, \quad (3)$$

in which case the growth rate no longer depends explicitly on  $\Delta'$  (Del Sarto et al. 2016). Here, barred quantities are normalized to the current sheet *thickness*.  $\bar{\tau}_A = a/V_A$  is the Alfvén crossing time,  $\bar{k} = ka$ , and the Lundquist number  $\bar{S} = \bar{\tau}_R/\bar{\tau}_A = aV_A/\eta_m$  where  $\bar{\tau}_R$  is the ohmic diffusion time over the thickness  $a$ . The expressions above may be used to find the scaling of the fastest-growing mode by assuming that both relations remain valid at the wavenumber of maximum growth  $k_m(\bar{S})$  for sufficiently large  $\bar{S}$ . For the Harris current sheet for which  $\Delta' \sim 2/ka$  this implies

$$\gamma\bar{\tau}_A \sim \bar{S}^{-\frac{1}{2}}, \quad \frac{\delta}{a} \sim \bar{S}^{-\frac{1}{4}}, \quad k_m a \sim \bar{S}^{-\frac{1}{4}}. \quad (4)$$

The relation for the “ideal” tearing instability, i.e., for an instability where the growth rate survives independently of the Lundquist number in the ideal limit (Pucci & Velli 2014), is obtained by rescaling the dispersion relation to the current sheet length rather than the thickness

$$\gamma\tau_A \sim S^{-\frac{1}{2}}\left(\frac{a}{L}\right)^{-\frac{3}{2}}. \quad (5)$$

Assuming an inverse aspect ratio of the form  $a/L \sim S^{-\alpha}$ , any value of  $\alpha < 1/3$  leads to a divergence of growth rates in the ideal limit, while any value of  $\alpha > 1/3$  leads to growth rates that tend to zero as the Lundquist number grows without bounds (Pucci & Velli 2014). This result is very general: any additional effect, such as viscosity (Tenerani et al. 2015a) or Hall current (Pucci et al. 2017), will result in a different critical aspect ratio scaling at which fast reconnection is triggered.

### 2.1. Modifications due to Ion–Neutral Interactions

In a partially ionized plasma, the effect of electron–neutral and electron–ion collisions on the plasma dynamics is the generation of an ohmic-type diffusion. In the presence of three different species undergoing collisions, the single-fluid description may apply in the partially ionized limit, with an appropriately modified magnetic induction equation.

Considering three different species (electrons, ions, and neutrals) the momentum conservation for each of the three species may be written separately, including interspecies collision terms, neglecting ionization and recombination effects. In Zweibel (1989) the Coulomb collisions between ions and electrons reflect in an ohmic diffusion coefficient in the induction equation that remains the same as in the fully ionized case. We notice here that, as shown in Singh & Krishan (2010), the actual value of the resistivity is enhanced if the electron–neutral collisions are taken into account, but the ohmic resistivity is substantially calculated in the same way, yielding a magnetic diffusivity

$$\eta_m = \frac{c^2}{\omega_{\text{pe}}^2}(\nu_{\text{ei}} + \nu_{\text{en}}), \quad (6)$$

where  $\omega_{\text{pe}}$  is the electron plasma frequency, the electron–ion and electron–neutral collision frequencies are  $\nu_{\text{ei, en}}$  and  $c$  is the speed of light. In Zweibel (1989) the interaction of the plasma

with neutrals occurs through ion–neutral collisions, while electron–neutral collisions are not taken into account. In this way, the tearing equation for the momentum conservation of ion and neutrals combined writes (primes denote derivatives with respect to the  $a$ -scaled variable  $y/a$ )

$$\begin{aligned} & (\gamma \bar{\tau}_{\text{Ai}})^2 \left( 1 + \frac{\nu_{\text{in}}}{\gamma + \nu_{\text{ni}}} \right) (\phi'' - \bar{k}^2 \phi) \\ &= -F(\psi'' - \bar{k}^2 \psi) + F'' \psi \\ & \psi = \bar{k} F \phi + \frac{1}{\bar{S} \gamma \bar{\tau}_{\text{Ai}}} (\psi'' - \bar{k}^2 \psi), \end{aligned} \quad (7)$$

and  $\bar{\tau}_{\text{Ai}}$  is the Alfvén time calculated with the ion density (still normalized to the sheet thickness  $a$ ),  $\gamma$  is the tearing growth rate associated with a mode with wavevector  $\bar{k} = ka$  along the equilibrium magnetic field. The collision frequencies are calculated assuming binary elastic (energy and momentum conservation) collisions between electrons and neutrals so that  $\nu_{\text{ni}} = \frac{n_i m_i}{n_n m_n} \nu_{\text{in}} \Rightarrow \nu_{\text{ni}} < \nu_{\text{in}}$  at most heights in the solar atmosphere (see Table 1 in Singh et al. 2015). Note that the opposite limit  $\nu_{\text{ni}} \gg \nu_{\text{in}}$  leads to the standard tearing of a completely ionized plasma. Following Zweibel (1989) we may redefine a starred Alfvén time and Lundquist number

$$\bar{\tau}_{\text{Ai}} \left( 1 + \frac{\nu_{\text{in}}}{\gamma + \nu_{\text{ni}}} \right)^{1/2} := \bar{\tau}_{\text{Ai}} f_M^{1/2} \rightarrow \bar{\tau}_{\text{A}}^*, \quad (8)$$

$$\bar{S}^* := \bar{S} \frac{\bar{\tau}_{\text{Ai}}}{\bar{\tau}_{\text{A}}^*}. \quad (9)$$

Inserting  $\bar{\tau}_{\text{A}}^*$  into Equation (7), and substituting  $\bar{S} \bar{\tau}_{\text{Ai}}$  with  $\bar{S}^* \bar{\tau}_{\text{A}}^*$  and  $\gamma \bar{\tau}_{\text{Ai}}$  with  $\gamma \bar{\tau}_{\text{A}}^*$ , the tearing mode equations regain their standard form, so that all the properties of the dispersion relation discussed previously now apply to the starred quantities. In Zweibel (1989) the modified tearing mode analysis is carried out only in the small  $\Delta'$  regime; see Equation (2). Here we analyze the tearing mode equations considering the maximum growth rate of the tearing instability of Equation (4), because the fastest-growing mode is the most relevant in the context of triggering fast magnetic reconnection in natural plasmas. In particular, from Equation (4) we have that  $\gamma \bar{\tau}_{\text{A}}^*$  follows the same scaling with  $\bar{S}^*$  as in the standard tearing theory:

$$\gamma \bar{\tau}_{\text{A}}^* \sim (\bar{S}^*)^{-1/2} \Rightarrow \gamma \bar{\tau}_{\text{Ai}} \sim (\bar{S})^{-1/2} \left( \frac{\bar{\tau}_{\text{Ai}}}{\bar{\tau}_{\text{A}}^*} \right)^{1/2}. \quad (10)$$

When the growth rate is negligible compared to both collision frequencies, the factor  $f_M^{1/2}$  becomes

$$f_M^{1/2} = \left( 1 + \frac{\nu_{\text{in}}}{\nu_{\text{ni}}} \right)^{1/2} = \left( 1 + \frac{\rho_n}{\rho_i} \right)^{1/2} = \left( \frac{\rho}{\rho_i} \right)^{1/2}, \quad (11)$$

where  $\rho = \rho_i + \rho_n$  is the total mass density. Introducing Equation (11) in (8) in this limit (growth rate negligible compared to both collision frequencies),  $\bar{\tau}_{\text{A}}^* = \bar{\tau}_{\text{A}}$ , i.e., the Alfvén time based on the Alfvén speed  $V_{\text{A}}$  calculated with the total (ion plus neutral) mass density. The Lundquist number  $\bar{S}^*$  also reduces to the Lundquist number based on the Alfvén speed calculated with the total density.

As in Zweibel (1989) and Singh et al. (2019), one may still define three different regimes for the maximum growth rate of the tearing mode including ion–neutral couplings. Though in Zweibel (1989) these are ordered by the magnitude of the growth rate relative to the neutral–ion and ion–neutral collision frequencies, it is better to provide an ordering based directly on the plasma parameters, since the growth rate of an instability depends exclusively on the scale lengths associated with the equilibrium, and it is the plasma parameters that determine the appropriate instability regime.

With the two ion–neutral collision frequencies, two intrinsic length scales are introduced into the resistive MHD equations that would otherwise remain scale free (that is why, in resistive MHD, it is the aspect ratio that appears as a crucial quantity defining current sheet instability). The length scales  $a_{c1, c2}$  are defined as

$$a_{c1, c2} = (\eta_m V_{\text{A}, \text{Ai}} / \nu_{\text{ni}, \text{in}}^2)^{1/3}. \quad (12)$$

These scales may be understood by comparing the growth rate of the fastest-growing tearing mode to the two collision frequencies,  $\nu_{\text{ni}, \text{in}}$ . From Equation (4), the fastest-growing tearing mode has a dimensional growth rate

$$\gamma = \frac{1}{(\bar{\tau}_{\text{R}} \bar{\tau}_{\text{A}})^{1/2}} = \left( \frac{\eta_m V_{\text{A}}}{a^3} \right)^{1/2}.$$

The tearing growth rate increases with shrinking current sheet thickness  $a$ . When starting from a thick sheet, the tearing mode will initially be so slow that the plasma will behave as a single fluid, with an Alfvén speed dictated by the total density. As the sheet thins and its thickness approaches  $a_{c1}$ , the growth rate approaches the frequency  $\nu_{\text{ni}}$ , when the ions and neutrals begin to decouple. As the sheet thins further, the growth rate continues to increase, and when the thickness decreases to  $a_{c2}$  the growth rate reaches  $\nu_{\text{in}}$ . At this point, the ions and neutrals are completely decoupled and the growth rate grows scaling only with the ionized plasma parameters.

Therefore, the scales determine the extent to which the ion–neutral couplings affect the dynamics of the problem. As detailed below, there are therefore three regimes: a coupled regime, for current sheets whose thickness  $a$  is larger than  $a_{c1}$ , for which the plasma behaves as a resistive fluid where the density is given by the total density; an intermediate regime,  $a_{c1} > a > a_{c2}$ , when there is partial coupling of the ions to neutrals; and an uncoupled regime for smaller scale sheets,  $a < a_{c2}$ , when the neutral and ion fluids decouple entirely. The corresponding tearing mode growth rates follow the same ordering, the growth rate increasing from one regime into the next as the scales decrease.

For each domain in current sheet thickness, we may define an appropriate timescale with which to normalize the growth rate. This is a matter of convenience, at this level, but becomes important later when taking the limit of very small resistivity (magnetic diffusivity) while keeping the ion–neutral collision frequencies finite. For the coupled regime, we will see that the natural timescale is the Alfvén time predicated on the total density. For the uncoupled regime, it is the timescale predicated on the ion density only. In the intermediate regime, we will show there is also an appropriate intermediate timescale.

*1. Coupled regime:*  $a \gg a_{c1}$ , i.e.,  $\gamma \ll \nu_{\text{ni}}$ : in this regime, Equation (10) simply means that the fastest tearing mode growth rate, normalized to the total density-based Alfvén time,

scales in the standard way with the total density-based Lundquist number (i.e., calculated using the Alfvén speed based on the total density and indicated now with subscript  $n$ ),  $\gamma\bar{\tau}_A \sim \bar{S}_n^{-1/2}$ . The result may also be written

$$\gamma\bar{\tau}_{Ai} \sim \bar{S}^{-1/2} \left( \frac{\rho}{\rho_i} \right)^{-1/4}. \quad (13)$$

Table 1 in Singh et al. (2015) shows that the ratio  $\rho_n/\rho_i$  can be up to  $10^6$  in some of the solar atmospheric layers. For such cases of interest the dispersion relation

$$\text{becomes } \gamma\bar{\tau}_{Ai} \sim \bar{S}^{-1/2} \left( \frac{\rho_n}{\rho_i} \right)^{-1/4}.$$

2. *Intermediate regime:*  $a_{c1} \gg a \gg a_{c2}$  or, equivalently,  $\nu_{ni} \ll \gamma \ll \nu_{in}$ : ion–neutral collisions only partially couple the ionized and neutral fluids, and the growth rate now scales as

$$\gamma\bar{\tau}_{Ai} \sim \bar{S}^{-1/2} \left( \frac{\nu_{in}}{\gamma} \right)^{-1/4} = \bar{S}^{-1/2} \left( \frac{\nu_{in}\bar{\tau}_{Ai}}{\gamma\bar{\tau}_{Ai}} \right)^{-1/4}, \text{ implying} \\ \gamma\bar{\tau}_{Ai} \sim \bar{S}^{-2/3} (\nu_{in}\bar{\tau}_{Ai})^{-1/3}. \quad (14)$$

Note, however, that we have normalized the growth rate here with the ion-based Alfvén time. A better way of writing this is

$$\gamma[\bar{\tau}_{Ai}(\nu_{in}\bar{\tau}_{Ai})] \sim [\bar{S}/(\nu_{in}\bar{\tau}_{Ai})]^{-2/3}, \quad (15)$$

showing that the appropriate normalization time for the growth rate is now the modified Alfvén time  $\bar{\tau}_{int} = \bar{\tau}_{Ai}(\nu_{in}\bar{\tau}_{Ai})$ , since it normalizes  $\gamma$  and redefines the Lundquist number  $\bar{S}_{int} = \bar{\tau}_R/\bar{\tau}_{int}$  in a homogeneous way with the same modified Alfvén time  $\bar{\tau}_{int}$ .

3. *Uncoupled regime:*  $a \gg a_{c2}$  or, equivalently,  $\gamma \gg \nu_{in}$ : ion–neutral collisions are too slow to couple the ionized and neutral fluids, so to lowest order  $\gamma\bar{\tau}_A^* \sim \gamma\bar{\tau}_{Ai}$ . Corrections of order  $\nu_{in}/\gamma$  can be found:

$$\gamma\bar{\tau}_{Ai} \sim \bar{S}^{-1/2} \left( 1 + \frac{1}{2} \frac{\nu_{in}}{\gamma} \right)^{-1/2} \sim \bar{S}^{-1/2} \left( 1 - \frac{1}{4} \frac{\nu_{in}}{\gamma} \right)$$

where  $\frac{\nu_{in}}{\gamma} := \epsilon \ll 1$  and we neglected terms of order  $\epsilon^2$ , leading to

$$\gamma\bar{\tau}_{Ai} \sim \bar{S}^{-1/2} - \frac{1}{4} \nu_{in}\bar{\tau}_{Ai}. \quad (16)$$

The ordering of the growth rate in the three regimes is completely equivalent to the corresponding ordering in the scales. This may be easily verified by direct substitution in the appropriate limiting cases, with the reminder that the ratio of the two critical thicknesses is  $a_{c1}/a_{c2} = (\rho_n/\rho_i)^{1/2} \gg 1$ .

In the next subsection we describe the initiation of reconnection within a framework of a dynamics driven by the corresponding fast normalizing timescale, making use of the results just obtained.

## 2.2. The Ideal Tearing Mode in Partially Ionized Plasmas

Following PV14, we now ask how thin a current sheet must become for its instability to be competitive with the typical dynamical timescale of the system, predicated now not on the thickness of the sheet but on a macroscopic length  $L$ , and therefore renormalizing all quantities using  $L$  in place of the equilibrium magnetic field scale  $a$ , i.e.,  $S^* = L/a\bar{S}^*$  and  $\tau_A^* = L/a\bar{\tau}_A^*$ .

Considering the ideal limit involves studying the asymptotics at large Lundquist numbers, i.e., small magnetic diffusivities,

and searching for the mode whose growth rate survives, but does not diverge, as  $\eta_m$  is allowed to go to zero while keeping the ion–neutral collisions finite. The limit means that both intrinsic scales  $a_{c1}$  and  $a_{c2}$  tend to 0, as does the current sheet thickness under study,  $a$  (via the aspect ratio  $a/L$ ). But the regime at which ideal tearing sets in will depend on the relative values of  $a$ ,  $a_{c1}$ , and  $a_{c2}$ .

The formal identity of the asterisked equations with the original tearing mode equations would lead to the renormalized dispersion relation

$$\gamma\tau_A^* \sim S^{*-1/2} \left( \frac{a}{L} \right)^{-3/2}. \quad (17)$$

Taking the limit of small resistivity while requesting the growth rate to remain finite would then lead to a solution

$$\gamma\tau_A^* \sim O(S^{*0} = 1) \quad (18)$$

with the critical current sheet thickness  $a_c$  and aspect ratio scaling

$$\frac{a_c}{L} \sim S^{*-1/3}. \quad (19)$$

This approach would seem to imply a normalization of the growth rate that depends on the growth rate itself. We have, however, already provided the solution to the full dispersion relation, as a function of the current sheet thicknesses  $a$ , in the previous section. So we can define, depending on which of the three regimes the critical sheet thickness falls in, the appropriate scale-independent timescale, i.e., renormalized with  $L$ . When  $a_c \gg a_{c1,c2}$ , the timescale will be the Alfvén time based on the total density,  $\tau_A = \bar{\tau}_A a/L$ ; when  $a_{c1} > a_c > a_{c2}$ , the timescale will be the intermediate timescale  $\tau_{int} = \bar{\tau}_{int} a/L$ , and when  $a_c \ll a_{c1,c2}$  the timescale becomes the shortest ion-only Alfvén time,  $\tau_{Ai} = \bar{\tau}_{Ai} a/L$ . This is very similar to what was done in Pucci et al. (2017), which deals with the effects of the Hall term on ideal tearing.

1. *Coupled regime:* In the coupled regime, as before, we assume that the critical aspect ratio, as defined by Equation (17), remains sufficiently large that  $a_c \gg a_{c1}$ . In this regime, then,  $V_A \ll L\nu_{ni}$ . When this is the case, we find an IT criterion based on the Lundquist number and Alfvén times based on the total (ion plus neutral) densities. This leads directly to the critical aspect ratio scaling

$$a_c/L \sim S_n^{-1/3} \simeq S^{-1/3} (\rho_n/\rho_i)^{1/6},$$

(recall that  $S$  is the Lundquist number based only on the ionic component). For the solar atmosphere the density dependence means the inverse aspect ratio can be up to 10 times larger than the fully ionized IT critical inverse aspect ratio (Singh et al. 2015).

2. *Intermediate regime:* If the inequalities identified above are not satisfied, then one enters the intermediate regime,  $V_{Ai} \ll L\nu_{in}$ , and in this regime,  $a_{c1} \gg a_c \gg a_{c2}$ . The renormalized dispersion relation now reads

$$\gamma\tau_{int} \sim S_{int}^{-2/3} (a/L)^{-2}, \quad (20)$$

where  $S_{int} = \tau_R/(\tau_{Ai}^2 \nu_{in}) = \bar{S}_{int}$  does not change on renormalization (both the resistive diffusion time and the Alfvén time in the denominator contain a length squared). Ideal tearing now

requires

$$\frac{a_c}{L} \sim S_{\text{int}}^{-1/3} = S^{-1/3}(\nu_{\text{in}}\tau_{\text{Ai}})^{1/3}, \quad (21)$$

$$\gamma_{\text{int}} = \gamma_{\text{Ai}}(\nu_{\text{in}}\tau_{\text{Ai}}) \sim 1 \rightarrow \gamma_{\text{Ai}} \sim (\nu_{\text{in}}\tau_{\text{Ai}})^{-1}. \quad (22)$$

The dependence of the aspect ratio on the Lundquist number is the same as the classical IT. The additional factor gives a slightly larger critical inverse aspect ratio scaling than in the fully ionized case, yet thinner than in the fully coupled regime. In this intermediate regime the critical current sheet remains thicker than in the fully ionized case.

**3. Uncoupled regime:** In this regime  $V_{\text{Ai}} \gg L\nu_{\text{in}}$ , the critical current sheet is very thin, i.e.,  $a_c \ll a_{c2}$ . The corrections to the standard IT tearing criterion depend only weakly on the small values of  $\nu_{\text{in}}\tau_{\text{Ai}}$ . The IT assumption now translates into  $\gamma_{\text{Ai}} \sim O(1)$ , so in this regime fast reconnection is triggered with the neutrals not really noticing.

### 2.3. The Inner Resistive Layer

The region around the neutral sheet, where the perturbations to the background field are significant, is the inner resistive layer  $\delta$  (see, e.g., Pucci et al. 2018). This parameter is particularly important for two different reasons: on the one hand, when  $\delta$  becomes of the order of the kinetic scales, kinetic effects play a role in the reconnection dynamics (see, e.g., Terasawa 1983; Pucci et al. 2017). On the other hand, previous work has shown that, at least in planar configuration, the reconnecting current sheet (if sufficiently long) disrupts in a series of self-similar steps, and  $\delta$  determines the thickness of the subsequent secondary current sheet thickness, and so on, recursively (Tenerani et al. 2015b). In Zweibel (1989) an estimation of  $\delta$  is given and the dependence on the ion–neutral collision frequency is recovered. In our case the expression for the maximum growth rate is given in Equation (4), where in the partially ionized case  $\delta/a \sim \bar{S}^{*-1/4} = (\tau_D/\tau_A^*)^{-1/4} = \bar{S}^{-1/4}f_M^{-1/8}$ . Since  $f_M$  is invariant for the IT rescaling, the solution is the same as for the classic IT with corrections depending on the regime,  $\delta/L = S^{-1/2}f_M^{-1/8}$ .

We can surmise that, as a current sheet thins in the solar atmosphere, though in the coupled regime the inner resistive layer is slightly larger than in the fully ionized case (Singh & Krishan 2010), the subsequent current sheets will rapidly transition to scales where reconnection is occurring only on the ionized component, and then down to kinetic effects. Future numerical simulations should confirm this result.

### 3. Summary and Conclusion

In this Letter we have discussed the onset of fast reconnection in partially ionized plasmas, considering three species undergoing collisions: ions, electrons, and neutrals. The ionization degree depends on the relative collision frequencies and we neglected the effect of ionization and recombination. Assuming as in Zweibel (1989) that the interaction with neutrals is dominated by binary ion–neutral collisions, we considered the combined ion and neutral equation of motion and the magnetic induction equation as the system describing the tearing instability of a generic equilibrium configuration. The magnetic diffusivity is also implicitly modified due to the additional collisions between neutrals and electrons. We derived the scalings for the tearing maximum growth rate for

three different regimes: coupled, intermediate, and decoupled, showing how the three regimes depend on current sheet thickness. We then calculated the inverse aspect ratio for which the growth rate does not depend on the Lundquist number.

In the coupled regime, the critical aspect ratio depends on the ratio between the neutral density and the ion density. The dependence is weak, but since  $\rho_n/\rho_i$  may be as large as  $10^6$  in the solar corona (Singh & Krishan 2010), the critical current sheet thickness can be up to 10 times larger than in the fully ionized case.




In the intermediate regime, the scaling with the Lundquist number remains the same as in the fully ionized case. A dependence on  $\nu_{\text{in}}\tau_{\text{Ai}}$  arises. However, the intrinsic thickness of the sheet remains thicker than in the decoupled regime, as shown by the inequalities between  $a_c$ ,  $a_{c1}$ , and  $a_{c2}$ .

Finally, in the decoupled regime a small correction ( $\sim \nu_{\text{in}}\tau_{\text{Ai}}$ ) arises with respect to the fully ionized case. This results in small corrections (factor  $<10$ ) to the critical aspect ratio.

On the basis of the above discussion one may outline the behavior of the tearing instability in a simple current sheet that is slowly thinning. At first, the tearing mode will develop on the global, ion–neutral coupled, Alfvénic timescale. Previous papers have shown that a recursive reconnection regime may appear (e.g., Tenerani et al. 2015b) that successively forms thinner sheets. These will transition to the intermediate and then fully decoupled regime, as the thicknesses of the sheets become thinner, accelerating the nonlinear evolution of the tearing mode.

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### References

- Alfvén, H. 1960, *AmJPh*, **28**, 613  
 Ballester, J. L., Alexeev, I., Collados, M., et al. 2018, *SSRv*, **214**, 58  
 Biskamp, D. 1986, *PhFl*, **29**, 1520  
 Comisso, L., Lingam, M., Huang, Y. M., & Bhattacharjee, A. 2016, *PhPl*, **23**, 100702  
 Del Sarto, D., Pucci, F., Tenerani, A., & Velli, M. 2016, *JGRA*, **121**, 1857  
 Furth, H. P., Killeen, J., & Rosenbluth, M. N. 1963, *PhFl*, **6**, 459  
 Huang, Y.-M., Comisso, L., & Bhattacharjee, A. 2017, *ApJ*, **849**, 75  
 Landi, S., Del Zanna, L., Papini, E., Pucci, F., & Velli, M. 2015, *ApJ*, **806**, 131  
 Landi, S., Papini, E., Del Zanna, L., Tenerani, A., & Pucci, F. 2017, *PPCF*, **59**, 014052

- Leake, J. E., Lukin, V. S., & Linton, M. G. 2013, [PhPI](#), **20**, 061202
- Leake, J. E., Lukin, V. S., Linton, M. G., & Meier, E. T. 2012, [ApJ](#), **760**, 109
- Loureiro, N. F., Schekochihin, A. A., & Cowley, S. C. 2007, [PhPI](#), **14**, 100703
- Parker, E. N. 1957, [JGR](#), **62**, 509
- Pucci, F., & Velli, M. 2014, [ApJL](#), **780**, L19
- Pucci, F., Velli, M., & Tenerani, A. 2017, [ApJ](#), **845**, 25
- Pucci, F., Velli, M., Tenerani, A., & Del Sarto, D. 2018, [PhPI](#), **25**, 032113
- Shibata, K., & Tanuma, S. 2001, [EP&S](#), **53**, 473
- Singh, K. A. P., Hillier, A., Isobe, H., & Shibata, K. 2015, [PASJ](#), **67**, 96
- Singh, K. A. P., & Krishan, V. 2010, [NewA](#), **15**, 119
- Singh, K. A. P., Pucci, F., Tenerani, A., et al. 2019, [ApJ](#), **881**, 52
- Sweet, P. A. 1958, [Obs](#), **78**, 30
- Tenerani, A., Rappazzo, A. F., Velli, M., & Pucci, F. 2015a, [ApJ](#), **801**, 145
- Tenerani, A., Velli, M., Rappazzo, A. F., & Pucci, F. 2015b, [ApJL](#), **813**, L32
- Terasawa, T. 1983, [GeoRL](#), **10**, 475
- Uzdensky, D. A., & Loureiro, N. F. 2016, [PhRvL](#), **116**, 105003
- Yamada, M., Kulsrud, R., & Ji, H. 2010, [RvMP](#), **82**, 603
- Zweibel, E. G. 1989, [ApJ](#), **340**, 550
- Zweibel, E. G., Lawrence, E., Yoo, J., et al. 2011, [PhPI](#), **18**, 111211
- Zweibel, E. G., & Yamada, M. 2009, [ARA&A](#), **47**, 291