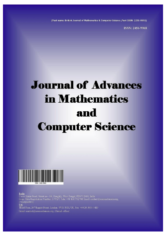

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On Some Representations of the Euler-Mascheroni Constants

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this study, we derive a new representation for the Euler-Mascheroni constant and present various expressions for the classical Euler-Mascheroni constant related to the Riemann zeta function. Also, we proved that γ is not algebraic if the Schanuel conjecture is true.

Keywords: Euler-Mascheroni constant; Schanuel Conjecture; Representations.

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1 Introduction

The Euler-Mascheroni constant or simply the Euler constant, γ, was first investigated by Euler in 1734 and later by Mascheroni. As a mathematical constant, the Euler-Mascheroni constant appears in the study of special functions such as the gamma and Riemann zeta functions and keeps recurring in number theory and analysis [1] observed that the Euler-Mascheroni constant is in the class of mathematical constants such as Ludolph's number (π) and Euler's number (e) and has applications in general theory of relativity and quantum theory.

It is known that π and e are irrational and transcendental, but little is known about the algebraic properties of the Euler-Mascheroni constant. An algebraic number is a complex number which is a root of a polynomial with rational coefficients and a transcendental number is a complex number which is not algebraic. It remains an open problem whether or not the Euler-Mascheroni constant is irrational, transcendental or algebraic.

Euler-Mascheroni constant, γ, is defined as

$$\gamma = \lim_{n\to\infty}\left(-\ln(n) + \sum_{j=1}^{n}\frac{1}{j}\right). \tag{1.1}$$

Now, other representations of the Euler-Mascheroni constant are given by

$$\gamma = \lim_{n\to\infty}\left(H_n - \ln(n)\right), \tag{1.2}$$

$$= \sum_{n=2}^{\infty}\frac{(-1)^n}{n}\zeta(n), \tag{1.3}$$

where H_n is the nth harmonic number and $\zeta(n)$ is the Riemann zeta function.

Euler introduced (1.1) and later Lorenzo Mascheroni also studied this constant and gave a numerical approximation for it up to 32 decimal places resulting in prominence as far as the Euler constant is concerned. [2] examined the Euler-Mascheroni constant, and described various mathematical developments about this constant and its connection with arithmetic functions, the Riemann hypothesis, random permutations, and random matrix products. In an essay on the Euler-Mascheroni constant, [3] delved into the recurrence of this constant in multiple branches of mathematics and intimated the possibility that, γ, could be transcendental. He also concluded that based on the computation of γ and its exponential, it strongly indicates that they are irrational.

[4] presented a new sequence that converges to the Euler-Mascheroni constant using an approximation of Pade type. They established lower and upper bound estimates between their sequence and the Euler-Mascheroni constant. A brief survey on the history of the Euler-Mascheroni constant, its applications and appearances in various mathematical settings appears in [5].

In this paper, a new series representation for the classical Euler-Mascheroni constant is derived through a generalized gamma function established in [6]. The study points to the connection between the Riemann zeta function and the Euler-Mascheroni constant.

2 Materials and Methods

The Riemann zeta is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, Re(s) > 1 \tag{2.1}$$

$$= \prod_{p} \left(1 - p^{-s}\right)^{-1}, Re(s) > 1 \tag{2.2}$$

where p is a prime number.

The derivatives of the Riemann zeta function is given by

$$\zeta^{(k)}(s) = (-1)^k \sum_{n=1}^{\infty} \frac{\ln^k n}{n^s}. \tag{2.3}$$

The digamma function is given by the logarithmic derivative of the gamma function:

$$\psi(z) = \frac{d \ln \Gamma(z)}{dz} = \frac{\Gamma^{"}(z)}{\Gamma(z)}. \tag{2.4}$$

The digamma function is defined as

$$\psi(z+1) + \gamma = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+z}\right), z \in \mathbb{C}, \tag{2.5}$$

where γ is the Euler-Mascheroni constant.

For $|z| < 1$,

$$\ln \Gamma(z+1) = -\gamma z + \sum_{k=2}^{\infty} \frac{(-1)^k \zeta(k)}{k} z^k, \tag{2.6}$$

$$= -\ln(1+z) - (\gamma - 1)z + \sum_{k=2}^{\infty} \frac{(-1)^k (\zeta(k) - 1)}{k} z^k. \tag{2.7}$$

Stirling numbers fascilitate expansion of $\ln \Gamma(z+1)$ in a power series about $z = 0$. The Stirling numbers of the first kind,$s(m, j)$, is defined by the generating function as

$$\ln^j \left(1 + \frac{z}{n}\right) = \sum_{m=j}^{\infty} \frac{j!}{m!} s(m, j) \left(\frac{z}{n}\right)^m, \tag{2.8}$$

where $|z| < 1$.

Alternatively, Stirling numbers of the first kind are also defined as

$$\frac{1}{j} \ln^j (1+t) = \sum_{n=j}^{\infty} s(n, j) \frac{t^n}{n!}, \tag{2.9}$$

or

$$\ln^j (1-t) = \sum_{n=j}^{\infty} \frac{(-1)^n}{n!} s(n, j) t^n. \tag{2.10}$$

From the above, we see that $s(n,1) = (-1)^{n-1}(n-1)!$, $s(n,n) = 1$,
$s(n,2) = (-1)^n (n-1)! \sum_{j=1}^{n-1} \frac{1}{j}$ and
$s(n,3) = \frac{1}{2}(-1)^{n+1}\left(H_{n-1}^2 - H_{n-1}^{(2)}\right)$,
where H_{n-1} is the $(n-1)$th harmonic number and $H_{n-1}^{(2)}$ is a harmonic number of order 2.

[6] established a generalized gamma function as

$$\Gamma_k(z) = \lim_{n\to\infty} \frac{\prod_{j=1}^{n} \exp\left(\frac{1}{k+1}\ln^{k+1}\left(1+\frac{1}{j}\right)^z\right)}{\exp\left(\frac{1}{k+1}\ln^{k+1} z\right)\prod_{j=1}^{n}\exp\left(\frac{1}{k+1}\ln^{k+1}\left(1+\frac{z}{j}\right)\right)} \tag{2.11}$$

and a functional equation as

$$\Gamma_k(z+1) = \exp\left(\frac{1}{k+1}\ln^{k+1} z\right)\Gamma_k(z), \tag{2.12}$$

where $z \in \mathbb{C}\setminus\mathbb{Z}^- U\{0\}$ and $k \in \mathbb{N}_0$.

Euler's formula for calculating $\zeta(2k)$ is expressed as

$$\zeta(2k) = \frac{(-1)^k (2\pi)^{2k} B_{2k}}{2(2k)!}, \tag{2.13}$$

where B_{2k} are Bernoulli numbers.

An identity for $\zeta(2k+1)$ was established in [7] as

$$\zeta(2k+1) = \frac{(-1)^{1-k}(2\pi)^{2k+1}}{2(2k+1)!}\int_0^1 B_{2k+1}(t)\cot(\pi t)dt, \tag{2.14}$$

where $k \in \mathbb{N}$.

[8] observes that if f_n is a sequence in a measurable function space L^+, then by the monotone convergence theorem

$$\int \sum_n f_n = \sum_n f_n \int f_n. \tag{2.15}$$

Lemma 2.1. *(Lindemann-Weierstrass)*

Given a positive integer n and distinct algebraic numbers $\alpha_0, ..., \alpha_n$. The numbers $e^{\alpha_0}, ..., e^{\alpha_n}$ are linearly independent over the set of algebraic numbers, A for $\beta_0, ..., \beta_n \in A$ not all zero.

Lemma 2.1 implies that

$$\sum_{k=0}^{n} \beta_k e^{\alpha_k} \neq 0. \tag{2.16}$$

Lemma 2.2. *(Schanuel Conjecture)*

Let $z_1 ..., z_n$ be complex numbers that are linearly independent over the rational numbers $\mathbb{Q}$. Then the extension field $\mathbb{Q}(z_1, ..., z_n, e^{z_1}, ..., e^{z_n})$ has transcendence degree of at least n where e^z is the complex exponential of z.

The Lemma below was established as a Corollary in [3].

Lemma 2.3. *For any positive integer n, the numbers $\ln\pi$ and $\sqrt{n}\pi$ are linearly independent.*

Remark 2.1. By letting $n = 1$ in lemma 2.3 implies that $\ln\pi$ and π are transcendental.

For $|z| < 1$, [6] established that

$$\ln^{k+1}(j+1) - \ln^{k+1} j = \frac{1}{j}(k+1)\ln^k j + \sum_{m=2}^{\infty}\left(\frac{1}{j}\right)^m \frac{1}{m!}\sum_{n=1}^{m}\frac{(k+1)!}{(k+1-n)!}s(m,n)\ln^{k+1-n} j. \quad (2.17)$$

In [9], the sine function is given as

$$\frac{\sin x}{x} = \prod_{n=1}^{\infty}\left(1 - \frac{x^2}{n^2\pi^2}\right). \quad (2.18)$$

3 Results and Discussion

We begin by establishing a new representation for the Euler-Mascheroni constant.

Theorem 3.1. *The Euler-Mascheroni constant is given by*

$$\gamma = \ln\pi - 2\sum_{m=2}^{\infty}\frac{\zeta(m)}{m2^m}, \quad (3.1)$$

where $\zeta(m)$ is the Riemann zeta function.

Proof. From (2.11), we have

$$\Gamma_k(z) = \frac{\exp\left(\sum_{j=1}^{\infty}\frac{z}{k+1}\ln^{k+1}\left(1+\frac{1}{j}\right)\right)}{\exp\left(\frac{1}{k+1}\ln^{k+1} z\right) * \exp\left(\sum_{j=1}^{\infty}\frac{1}{k+1}\ln^{k+1}\left(1+\frac{z}{j}\right)\right)}. \quad (3.2)$$

By (2.17), we obtain

$$\Gamma_k(z) = \frac{\exp\left(\sum_{m=2}^{\infty}\left(\frac{z}{m!}\right)\sum_{n=1}^{m}\frac{k!s(m,n)}{(k+1-n)!}\zeta^{(k+1-n)}(m)\right)}{\exp\left(\frac{1}{k+1}\ln^{k+1} z\right) * \exp\left(\sum_{m=2}^{\infty}\left(\frac{z^m}{m!}\right)\sum_{n=1}^{m}\frac{k!s(m,n)}{(k+1-n)!}\zeta^{(k+1-n)}(m)\right)}. \quad (3.3)$$

By letting $z = -\frac{1}{2}$ and $k = 0$ yields

$$\Gamma\left(-\frac{1}{2}\right) = \frac{\exp\left(-\frac{1}{2}\sum_{m=2}^{\infty}\left(\frac{1}{m!}\right)\sum_{n=1}^{m}\frac{s(m,n)}{(1-n)!}\zeta^{(1-n)}(m)\right)}{\exp\left(\ln\left(\frac{-1}{2}\right)\right)\exp\left(\sum_{m=2}^{\infty}\left(\frac{\left(-\frac{1}{2}\right)^m}{m!}\right)\sum_{n=1}^{m}\frac{s(m,n)}{(1-n)!}\zeta^{(1-n)}(m)\right)},$$

$$= \frac{\exp\left(-\frac{1}{2}\sum_{m=2}^{\infty}\left(\frac{1}{m!}\right)\sum_{n=1}^{m}\frac{s(m,n)}{(1-n)!}\zeta^{(1-n)}(m)\right)}{\exp\left(\ln\left(\frac{i^2}{2}\right)\right)\exp\left(\sum_{m=2}^{\infty}\left(\frac{\left(-\frac{1}{2}\right)^m}{m!}\right)\sum_{n=1}^{m}\frac{s(m,n)}{(1-n)!}\zeta^{(1-n)}(m)\right)},$$

$$-2\sqrt{\pi} = -2\exp\left(\frac{1}{2}\sum_{m=2}^{\infty}\frac{(-1)^m\zeta(m)}{m} + \sum_{m=2}^{\infty}\frac{(-1)^{2m}\zeta(m)}{m * 2^m}\right).$$

Further simplifying gives

$$\sqrt{\pi} = \exp\left(\frac{\gamma}{2} + \sum_{m=2}^{\infty} \frac{\zeta(m)}{m2^m}\right). \tag{3.4}$$

By applying logarithm on both sides of (3.4) and making γ the subject completes the proof. □

Remark 3.1. If $k = 0$ and $z = \frac{1}{2}$ in (3.3), we obtain

$$\Gamma\left(\frac{1}{2}\right) = 2\exp\left(-\frac{1}{2}\sum_{m=2}^{\infty} \frac{(-1)^m\zeta(m)}{m} + \sum_{m=2}^{\infty} \frac{(-1)^m\zeta(m)}{m * 2^m}\right). \tag{3.5}$$

Simplifying further gives

$$\sqrt{\pi} = 2\exp\left(\frac{-\gamma}{2} + \sum_{m=2}^{\infty} \frac{(-1)^m\zeta(m)}{m2^m}\right). \tag{3.6}$$

Taking logarithm on both sides of (3.6) and making γ the subject gives

$$\gamma = \ln\frac{4}{\pi} + 2\sum_{m=2}^{\infty} \frac{(-1)^m}{m2^m}\zeta(m), \tag{3.7}$$

which is a known representation of the Euler-Mascheroni constant also found in [10].

We observe that the series in (3.1) converges faster than that of the Euler-Mascheroni constant given by (1.3). The Euler-Mascheroni constant, as a mathematical constant, keeps reoccurring in the study of special functions and in analysis, supportive of the suitability of the constant $\lambda = \sum_{m=2}^{\infty} \frac{\zeta(m)}{m*2^m}$ in numerical calculations where γ converges slowly.

By the use of infinite series calculator (www.wolframalpha.com/widgets), the constant λ converged to 0.283757.

Next, we present a new representation of the constant, e^γ, in corollary 3.2.

Corollary 3.2.

$$e^\gamma = \frac{\pi}{e^{2\lambda}}, \tag{3.8}$$

where γ is the Euler-Mascheroni constant and $\lambda = \sum_{m=2}^{\infty} \frac{\zeta(m)}{m*2^m}$

Proof. From (3.1), we obtain

$$\pi = e^\gamma e^{2\sum_{m=2}^{\infty} \frac{\zeta(m)}{m*2^m}}. \tag{3.9}$$

By making e^γ the subject in (3.9) completes the proof. □

The constant e^γ is vital in number theory and relates to the Marten's third theorem in the following form:

$$e^\gamma = \lim_{n\to\infty} \frac{1}{\ln p_n} \prod_{i=1}^{n} \frac{p_i}{p_i - 1}, \tag{3.10}$$

where p_n is the nth prime number.

Following Theorem 3.1, we present various representations of the Euler-Mascheroni constant.

Theorem 3.3.

$$\gamma = \ln\left(\ln(i)^{-2i}\right) - 2\sum_{m=2}^{\infty}\frac{\zeta(m)}{m2^m}, \tag{3.11}$$

where $i^2 = -1$.

Proof. Given Euler's identity, we obtain

$$\pi = \ln(i)^{-2i}. \tag{3.12}$$

Substituting (3.12) into (3.1) completes the proof. □

Theorem 3.4.

$$\gamma = \ln\frac{1}{2} - 2\zeta'(0) - 2\sum_{m=2}^{\infty}\frac{\zeta(m)}{m2^m}. \tag{3.13}$$

Proof. [11] established that

$$\zeta^{'}(0) = -\frac{1}{2}\ln 2\pi. \tag{3.14}$$

By making $\ln\pi$ the subject, we get

$$\ln\pi = -\ln 2 - \zeta'(0). \tag{3.15}$$

Substituting (3.15) into (3.1) completes the proof. □

Theorem 3.5.

$$\gamma = \ln\frac{1}{2} + 2\sum_{m=2}^{\infty}\left(\ln m - \frac{\zeta(m)}{m2^m}\right). \tag{3.16}$$

Proof. For $k = 1$ and $z = 0$, (2.3) becomes

$$\zeta'(0) = -\sum_{m=1}^{\infty}\ln m. \tag{3.17}$$

which yields

$$-\frac{1}{2}\ln 2\pi = \sum_{m=1}^{\infty}\ln m. \tag{3.18}$$

Making $\ln\pi$ the subject, we obtain

$$\ln\pi = \ln\frac{1}{2} + \sum_{m=1}^{\infty}\ln m^2. \tag{3.19}$$

Substituting (3.19) into (3.1) completes the proof. □

Theorem 3.6.

$$\gamma = \ln\sqrt{6} + \frac{1}{2}\ln\prod_{p}\left(1 - p^{-2}\right)^{-1} - 2\sum_{m=2}^{\infty}\frac{\zeta(m)}{m2^m}. \tag{3.20}$$

Proof. Substituting $s = 2$ into (2.1) gives

$$\zeta(2) = \prod_p \left(1 - p^{-2}\right)^{-1}, \tag{3.21}$$

and further yields

$$\frac{\pi^2}{6} = \prod_p \left(1 - p^{-2}\right)^{-1}. \tag{3.22}$$

Applying logarithm on both sides of (3.22), we get

$$\ln \pi = \frac{1}{2} \ln 6 + \frac{1}{2} \ln \left(\prod_p \left(1 - p^{-2}\right)^{-1} \right). \tag{3.23}$$

Substituting (3.23) into (3.1) ends the proof. □

Remark 3.2. Comparing (3.13) and (3.23), we get

$$\zeta^{'}(0) = \ln \left(\frac{\sqrt{\sqrt{6}}}{2\sqrt{3}} \right) - \frac{1}{4} \ln \left(\prod_p \left(1 - p^{-2}\right)^{-1} \right). \tag{3.24}$$

Theorem 3.7.

$$\gamma = \ln 2 + \sum_{m=1}^{\infty} \ln \left(\frac{4m^2}{4m^2 - 1} \right) - 2 \sum_{m=2}^{\infty} \frac{\zeta(m)}{m 2^m}. \tag{3.25}$$

Proof. By letting $x = \frac{\pi}{2}$ and substituting into (2.18) , we obtain

$$\frac{\pi}{2} = \ln \left(\prod_{m=1}^{\infty} \left(\frac{2m}{2m-1} \frac{2m}{2m+1} \right) \right), \tag{3.26}$$

a Wallis product formula for $\frac{\pi}{2}$. □

Taking logarithm on both sides of (3.26) gives

$$\ln \pi = \ln 2 + \ln \left(\prod_{m=1}^{\infty} \left(\frac{2m}{2m-1} \frac{2m}{2m+1} \right) \right). \tag{3.27}$$

Substituting (3.27) into (3.1) ends the proof.

Theorem 3.8. *The constant λ is given by*

$$\lambda = \sum_{m=1}^{\infty} \left(\ln m - \ln \left(m - \frac{1}{2} \right) - \frac{1}{2m} \right). \tag{3.28}$$

Proof. Integrating (2.5) and using (2.15), we get

$$\ln \Gamma(z+1) + \gamma z = \sum_{m=1}^{\infty} \left(\frac{z}{m} - \ln(m+z) + \ln(m) \right). \tag{3.29}$$

By letting $z = -\frac{1}{2}$, (3.33) yields

$$\ln \pi - \gamma = 2 \sum_{m=1}^{\infty} \left(-\frac{1}{2m} - \ln \left(m - \frac{1}{2} \right) + \ln m \right). \tag{3.30}$$

By substituting (3.30) into (3.1) completes the proof. □

New series representations emerge for the Euler-Mascheroni constant involving Bernoulli numbers and Bernoulli polynomials.

Theorem 3.9.

$$\gamma = \ln \pi - \sum_{k=1}^{\infty} \frac{(-1)^k (\pi)^{2k} B_{2k}}{2k(2k)!} + \sum_{k=1}^{\infty} \frac{(-1)^{1-k} (\pi)^{2k+1}}{(2k+1)(2k+1)!} \int_0^1 B_{2k+1}(t) \cot(\pi t) dt, \tag{3.31}$$

where B_{2k} and $B_{2k+1}(t)$ are Bernoulli numbers and polynomials respectively.

Proof. For even values of m, (3.1) becomes

$$\gamma = \ln \pi - 2 \left(\sum_{m=2}^{\infty} \frac{\zeta(m)}{2^m m} - \sum_{m=2}^{\infty} \frac{\zeta(m+1)}{2^{m+1}(m+1)} \right). \tag{3.32}$$

Let $m = 2k$ for $k \in \mathbb{N}$, then we obtain

$$\gamma = \ln \pi - 2 \left(\sum_{k=1}^{\infty} \frac{\zeta(2k)}{2^{2k} 2k} - \sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{2^{2k+1}(2k+1)} \right). \tag{3.33}$$

Substituting (2.13) and (2.14) into (3.33), the proof is complete. □

In Theorem 3.1, $\ln \pi$ is a term in the new representation for the Euler-Mascheroni as well as the constant $\lambda = \sum_{m=2}^{\infty} \frac{\zeta(m)}{m 2^m}$. Understanding the nature of these constants can reveal the nature of the Euler-Mascheroni constant. Already, it is observed that the constant λ is algebraic in nature. The transcendentality of $\ln \pi$ is shown in lemma 3.10.

Lemma 3.10. *Let the Schanuel conjecture be true. Then, $\ln \pi$ is transcendental.*

Proof. Assuming lemma 2.2 and lemma 2.3 are true. This implies that π and $\ln \pi$ are algebraically independent over the set of rational numbers.

Thus, if lemma 2.2 and lemma 2.3 are true, then $\ln \pi$ is transcendental. □

Claim

γ is transcendental.

Proof. By (3.1) and Lemma 3.10, the proof is complete. □

Remark 3.3. The Euler-Mascheroni constant is not algebraic.

4 Conclusions

A new series representation of the Euler-Mascheroni constant has also been derived and various representations given. We also present a new representation for e^γ which is useful in number theory and proved that if $\ln \pi$ is transcendental, then the Euler-Mascheroni constant is also transcendental.

Competing Interests

Authors have declared that no competing interests exist.

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