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Fitting Rwanda's Currency Market Returns with the Poisson Compound Model with Normal Inverse Gaussian Jumps

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Abstract

The first step towards developing a reliable forecast for the forex market returns is to find a model that can be used and explains all of the return behavior. The compound Poisson model with the Normal Inverse Gaussian jumps (NIG), geometric Brownian motion (gBm), and the Poisson model with the Gaussian jumps (Norm) are used in this work to fit the market data. The AIC and BIC scores of the models were used to validate them. The model compound Poisson with Normal Inverse Gaussian jumps (NIG) performs better across all currencies than the model Norm and gBm. The data from the Rwandan forex market matches well with a model compound Poisson with Normal Inverse Gaussian jumps (NIG).

Keywords

Geometric Brownian Motion (gBm), Forex, Compound Poisson Model with Gaussian Jumps (Norm), Compound Poisson Model with Normal Inverse Gaussian Jumps (NIG)

1. Introduction

The Brownian movement model and the Compound Poisson measure are both present in a focused class of stochastic cycles shaped by the Levy process. Applications for these models range from material science and science to money and protection. They allow for the depiction of unexpected moves by hops, similar to the Poisson cycle, which is an essential component for some applications. Over the past ten years, Levy measures have been thought to have expanded significantly as they are discussed from both a scholarly perspective, where they provide stochastic models of money-related business sectors, and a financial one, where they are informed by a variety of amazing alumni communications. This keeps inspiring more research in both practical and theoretical contexts (Ayed, Lee, & Caron, 2020).

Levy measures have garnered significant attention in the field of numerical accounting in recent times, largely due to their diverse applications. As a result, they are now considered an essential building element for the presentation of foreign currency. Toll cycle models should be considered first in developing and evaluating quantifiable procedures, as they capture the most important aspects of market returns and serve as "first-request approximations" to other, more precise models. It should be noted that the combination of a geometric Brownian and a compound Poisson measure yields the broadest Levy measure (Broadie & Kaya, 2006).

All of the market return's behavior can be explained by these models, which is good news. To represent more customized highlights of market returns, other Levy-based models have recently been proposed. Retail currency traders use forex analysis to decide which currency pairs to buy and sell. Gaining an understanding of the macroeconomic principles governing currency value could make currency traders more successful. In order to do this, traders can fix exchange rates, buy and sell currencies in advance on futures swaps, or buy and sell them in futures swap markets. Users can profit from the appreciation or devaluation of various currencies by using "How to trade the Forex market" and "Forex trading" (Ennals, 2018).

The difference in the valuations of currencies between traders and buyers has shrunk along with foreign exchange spreads (buyers have a broader focus on the long-term ownership and use of financial instruments). Because of this, the value of different currencies fluctuates, necessitating the use of foreign exchange services and commerce. The exchange rate results in a higher value for the original currency, which benefits investors (Garner, 2021).

In Teneng (2013b), Dean Teneng predicted foreign exchange rates using three models. Teneng proved that the guileless Random Walk model is outperformed by the conjecture of unknown trade day-by-day shutting costs using the standard reverse Gaussian (NIG) and Variance Gamma (VG) Toll measures. The models NIG and Norm were not both used in this paper. Furthermore, it was not attempted to fit all three models for every cash and determine which one performs better.

This work (Teneng, 2013a) in order to trade shutting costs, this work focused on fitting the Typical Inverse Gaussian (NIG) circulation using the open programming bundle R and selecting the least complex models using the method suggested by Käärik and Umbleja. The daily shutting costs (12/04/2008-07/08/2012) of NZD/USD, QAR/CHF, QAR/EUR, CHF/JPY, SAR/CHF, AUD/JPY, GBP/JPY, SAR/EUR, TND/CHF, and TND/EUR tend to be magnificent fits, whereas EGP/EUR and EUR/GBP are acceptable fits, with a Kolmogorov-Smirnov test with p-worth of 0.062 and 0.08 for each. This means that the daily shutting costs of CHF/JPY, GBP/JPY, QAR/EUR, SAR/EUR, TND/CHF, EGP/EUR, EUR/GBP, and TND/EUR (12/04/2008-07/08/2012) have been consistently illustrated using ordinary opposite Gaussian circulation, and their future costs have been gauged using NIG Lévy measure. He didn't use any in this paper.

In Reddy and Clinton (2016), Reddy and Clinton look into the Geometric Brownian development model as a means of simulating stock value and provide three methods for evaluating the model's validity. Daily stock value data from January 1, 2013, to December 31, 2014, was obtained from the Thomson One data set. Because of their year-based limitations, there is a significant discrepancy in the estimation in this paper, making it generally unreliable.

The study (Mingola, 2013) demonstrates how many radioactive particles a model produces. The amount of α-particles delivered up to time t can be represented by a Poisson cycle, $\{N(t): t \geq 0\}$ with force λ, given an enormous variety of radioactive centers that transmit α-particles on schedule and for times impressively not the half presence of the radioactive substance (this can go from a little section of a second to billions of years). A Poisson association can demonstrate customer purchasing attentiveness; a Poisson cycle can also demonstrate the Coupon Collection difficulty; and a Poisson cycle can also demonstrate shot upheaval.

The research (Agustini, Affianti, & Putri, 2018) suggests that the Mathematical Brownian evolution could be used as a numerical model to predict the stock's longer-term cost. The step that comes before estimating stock value is calculating the stock expected value plan and selecting the 95% assumption level. The calculation of stock value expectation using the mathematical Brownian development model starts with determining the value of return, followed by evaluating the value of float and volatility, obtaining an estimate of stock value, calculating the gauge MAPE, registering the stock expected cost, and calculating the 95% egotism level. Supported by the analysis, the yield study demonstrates that the high-accuracy expectation process is the mathematical Brownian development model. It is illustrated using an approximated MAPE.

The research (Shaw, 2019) presents a framework that exchanges the Levy model to examine changes in power Option-Adjusted Spreads (OAS), which are the spreads between a spot Treasury wind and an enrolled OAS record of all bonds in a particular rating group. Every constituent security's OAS is used to compile an OAS record, which is weighted by market capitalization. OAS monitors a degree of credit risk in bonds that are selected. These are interesting for particular reasons. The idea that corporate security yields indicate the cost of financing for private companies is one such reason. Excessive spreads indicate higher capital costs and, hence, lower experience opening upside. Additionally, given that the premium on genuine capital is a major factor in cash-related.

This paper makes the following contributions: 1) Fitting the returns on the Rwandan forex market with the Geometric Brownian motion 2) Using a compound Poisson model with Gaussian jumps to fit the returns of the 3 Rwandan forex market. 3) Using a compound Poisson model with normal inverse Gaussian jumps to fit the returns of the Rwandan forex market. 4) Use both AIC and A measures to compare each model's performance.

2. Methodology

2.1. Description of the Models Used

The inverse Gaussian distribution model, which finds widespread use in intricate applications, is the inverse of a single continuous normal distribution. As discussed in this paper (Schrodinger, 1915), Schrödinger initially deduced this kind of distribution in 1915 when researching the Brownian movement for the first time. Étienne Halphen was the one who suggested this distribution (Banks, 1999). In 1945, Tweedie's inverse Gaussian name was approved (Folks & Chhikara, 1978). The inverse normal distribution models depict an exponentially composite model with a long tail to the right and only one pattern. One of the distributions used in the generalized linear model tracking process is this one (Armitage, 1950). In addition to its many varied applications in economics and business, survival analysis, finance, medical, and even labor dispute settlement, this distribution is employed to track data with positive deviation (Tweedie, 1957).

In an effort to follow Brownian motion in physics, the normal inverse Gaussian distribution model was initially examined in 1956. Because there is an inverse link that expresses the time needed to cover the unit's distance and the distance covered at the time the unit was approved, M.C.K. Tweedie originally utilized it as an inverse Gaussian (Chhikara, 1988). A variance-mean mixture of a normal distribution, with the inverse Gaussian serving as the mixing distribution, is known as the normal inverse Gaussian distribution. The inverse Gaussian process subordinates Brownian motion to a homogeneous Lévy process, which can be represented (Barndroff-Neielsen, 1997). The normal inverse Gaussian (NIG) distribution was put up by (Barndorff-Nielsen et al., 1998) as a potential stock price model. An alternative representation of this process would be a time-changed Brownian motion, in which the first passage time of an independent Brownian motion with drift to the level t is denoted by the time change $T(t)$. Therefore, the time change is an inverse Gaussian process. This proposes the nomenclature of a regular inverse Gaussian process when one analyzes a Brownian motion at this point (Lahcene, 2019).

2.2. Definition: Levy Process

With values in R^d, a stochastic interaction $(X_t)_{t\geq 0}$ on (Ω, F, P) to the extent that X_0 is described as a Levy Process if it possesses the following characteristics:

1) Stationary increments: t is independent of the law of $X_{t+h} - X_t$.

2) Self-governing augmentations: for every growing grouping of times $t_0, \cdots, t_n$, the abnormal $X_{t_0}, X_{t_1} - X_{t_0}, \cdots, X_{t_n} - X_{t_{n-1}}$ variables are Autonomous.

3) Probabilistic congruence: $\forall \epsilon > 0$, $\lim_{h \to \infty} \left[\left| X_{t+h} - X_t \right| \geq \epsilon \right] = 0$

4) Most likely, it's càdlàg.

5) $X_0 = 0$ Without a question

The third condition implies that for a given time frame t, the likelihood of seeing a seize t is zero: discontinuities consistently happen indiscriminately times (Huang, 2008). Suppose H_t is a Levy cycle. At that point, K_t, the interaction, is described as

$$K_t = K_0 e^{H_t}, t > 0. \tag{1}$$

Equation (1) is known as an extraordinary Levy measure. When autonomous log returns are possible, this cycle is typically used to display resource measures. Certainly, in the unlikely event that we use the technique's log-returns K_t

$$\log \left[K_{t+\Delta t} / K_t \right] = H_{t+\Delta t} - H_t = \Delta H_t. \tag{2}$$

Equation (2) is referred to as an exceptional Levy measure. Usually, resource measures are displayed using this cycle when autonomous log returns are possible. Of course, in the improbable case that we employ the method's log-returns (Iacus & Yoshida, 2018).

2.3. Generalities on the Gaussian Distribution

2.3.1. Inverse Gaussian Process

The Gaussian distribution that is inverse, the distribution $\mathrm{IG}(\delta, \gamma) = \mathrm{GIG}(-1/2, \delta, \gamma)$ has the probability density given by Equation (3) below:

$$p_{\mathrm{IG}}(x; \delta, \gamma) = \left(\frac{\delta^2}{2\pi} \right)^{\frac{1}{2}} x^{-\frac{3}{2}} \exp\left[-\frac{\delta^2 \left(x - \delta\gamma^{-1} \right)^2}{2\left(\delta\gamma^{-1} \right)^2 x} \right], \tag{3}$$

for $x > 0, \delta > 0$ and $\gamma > 0$. More broadly, the Equation (4) of its Fourier transform indicates that the function $p_{\mathrm{IG}}(\cdot; \delta, \gamma)$ specifies a probability distribution.

Consequently, the following replicating property is satisfied by the class of inverse Gaussian distributions: The expression $\varphi_{\mathrm{IG}}(u; \delta, \gamma) = \int_0^\infty e^{iux} p_{\mathrm{IG}}(x; \delta, \gamma) \mathrm{d}x$:

$$\varphi_{\mathrm{IG}}(u; \delta, \gamma) = \exp\left[\gamma\delta \left(1 - \sqrt{1 - 2iu/\gamma^2} \right) \right], u \in R. \tag{4}$$

The equivalent Lévy process for $\mathrm{IG}(\delta_1, \gamma) * \mathrm{IG}(\delta_2, \gamma) = \mathrm{IG}(\delta_1 + \delta_2, \gamma)$ is known as an inverse Gaussian process with parameters $\mathrm{IG}(\delta, \gamma)$. It is evident that $\mathrm{IG}(\delta, \gamma)$ is infinitely divisible with parameters $\delta > 0$ and $\gamma > 0$ (Iacus & Yoshida, 2018).

2.3.2. Normal Inverse Gaussian Process

Barndoff-Neilson introduced the NIG conveyance cycle as a log-returns of stock cost model. It belongs to the wider category of exaggerated Levy measures. Following its presentation, it became evident that the NIG dispersion fits log-returns of securities exchange data the best. Additional analyses have demonstrated that, in comparison to other resource classes, this appropriation exhibits tantrums. Below is a description of the density function: $\text{NIG}(\alpha,\beta,\delta,\mu)$

$$f_{\text{NIG}}(x;\alpha,\beta,\delta,\mu)=\frac{\alpha}{\pi}\text{e}^{\delta}\sqrt{\alpha^2-\beta^2+\beta(x-\mu)\frac{K_1\left(\alpha\delta\sqrt{1+(x-\mu)^2/\delta^2}\right)}{\sqrt{1+(x-\mu)^2/\delta^2}}}, \quad (5)$$

where $\delta>0$, $\alpha>0$. The parameters of the Normal Inverse Gaussian transport in Equation (5) are as follows: α represents the tail steepness, β the skewness, δ the scale, and μ the area. The NIG dispersion is the only member of the set of generalized heightened distributions to be closed under convolution (Shaw, 2019).

An inverse Gaussian process provided by $K=(K_t)_{t\in R_+}$ with $\mathcal{L}\{K_1\}=\text{IG}\left(\delta,\sqrt{\alpha^2-\beta^2}\right)$ where $\delta,\alpha\in(0,\infty)$, $\beta\in R$ Satisfying $\alpha\ge|\beta|$; $\gamma=\sqrt{\alpha^2-\beta^2}$. Define $X=(X_t)_{t\in R_+}$ by $X_t=t\mu+K_t\beta+W_{tk}$, with $\mu\in R$ and $W=(W_t)_{t\in R_+}$ being a one-dimensional wiener process independent of *K*. Let $\beta K+W_k$ be the inverse Gaussian process *K*, which subordinates a one-dimensional drifting Wiener process $Y_k=\beta k+W_k$. Consequently, *X* and $Y_k=\beta k+W_k$ are Lévy process. Let $\text{NIG}(\alpha,\beta,\delta,\mu)$ be the normal inverse Gaussian process for the process *X*. We define the distribution as follows:

$$\mathcal{L}\{X_1\}=\mathcal{L}\{\mu+\beta K_1+W_{k_1}\} \quad (6)$$

Equation (6) is known as a normal inverse Gaussian distribution $\text{NIG}(\alpha,\beta,\delta,\mu)$. Through

$$E\left[\exp\left(iu\left(\beta k_1+W_{k_1}\right)\right)\right]=E\left[\exp\left(iu\left(\beta+2^\wedge\{-1\}iu\right)k_1\right)\right]$$

with the help of Equation (4) we can determine the characteristics function given in Equation (7)

$$\varphi_{\text{NIG}}(u;\alpha,\beta,\delta,\mu)=\exp\left[iu\mu+\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(\beta+iu)^2}\right)\right], \quad (7)$$

(Iacus & Yoshida, 2018).

2.4. Generalities Standard Processes

2.4.1. Geometric Brownian Motion

A Brownian (or Wiener) process is the stochastic process $B=B_t$: $t>0$ if and only if

1) When $s<t$, the increment B_t-B_s is independent of the history of the process's actions up to time *s*. Thus, B_t has independent increments;

2) Gaussian increments characterize B_t; thus, the distribution $B_t-B_s\sim N(0,t-s)$.

3) Although B_t is continuous, it is not smooth, which means it cannot be differentiated anywhere.

4) $B_0 = 0$.

In the cosmic system of stochastic cycles used to demonstrate value changes, the Brownian movement is without a doubt the most brilliant star. A Brownian movement is a random measure W_t with autonomous, fixed additions that follow a Gaussian circulation. Brownian movement is the most generally contemplated stochastic measure and the mother of the cutting-edge stochastic examination (Neisy, 2017). The Brownian movement also, monetary displaying has been integrated from the earliest starting point of the last mentioned, when Louis Bachelier proposed to show the value K_t of an asset at the Paris Bourse as (Tankov & Cont, 2003): $K_t = K_0 + \sigma W_t$. The expression "Brownian movement" can likewise allude to a cycle W_t which fulfills the primary, second, and fourth conditions above, yet which has dissemination for $W_t - W_s = N\left(\mu(t-s),(t-s)\sigma^2\right)$, where μ is named the floating coefficient and σ the dispersion coefficient. The connection between standard Brownian movement and Brownian movement is equivalent to that among $N(0, 1)$ and $N(\mu t, \sigma^2 t)$ (Morters & Peres, 2010).

The model for fitting currency market returns could then be geometric Brownian motion

$$\frac{dK_t}{K_t} = \mu dt + \sigma dB \tag{8}$$

where $dB \sim N(0, dt)$ and μ is the drift and σ is the volatility, and a starting value $K_0 > 0$ in Equation (8). What is more, is an exceptional instance of this model as its answers are given inside the structure

$$K_t = K_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right], \tag{9}$$

Occasionally, the process K_t in Equation (9) is referred to as a geometric Brownian motion (Garrett, 2013).

2.4.2. Compound Poisson Process

For financial applications, A cycle with only one potential leap size is not very interesting. The leap sizes may exhibit a self-emphatic movement in the compound Poisson measure, where the holdings up events between hops are exceptional in any case. Alternatively, consider N as a Poisson cycle with boundary λ and $\{Y_i\}_{i\geq 1}$ be an assembly of independent, self-assured parts (Tankov, 2007). Stochastic process X_t is defined as a compound Poisson process with intensity $\lambda > 0$ and, hence, the jump size distribution f,

$$X_i = \sum_{i=1}^{N_t} Y_i. \tag{10}$$

In this Equation (10), N_t is a Poisson method with intensity λ, which makes it independent from $(Y_i)_{i\geq 1}$. The jump sizes, Y_i, are independent and identically distributed (i.i.d) with distribution $3f$.

The definition of a compound Poisson method yields the following properties, which can be readily ascertained:

1) The sample paths of X are piecewise constant functions.

2) $(T_i)_{i\geq 1}$ is the jump times with the same law as the jump times of the Poisson process N_t: they can take the form of partial sums of free exponential random variables with parameter λ.

3) These jump sizes $(Y_i)_{i\geq 1}$ are independent and identically distributed (i.i.d.) with law f (Tankov & Cont, 2003).

The Poisson interaction (the leap part) and Brownian motion (the dissemination part) are the two fundamental structure squares of every Lévy cycle. Since the Black-Scholes model is present, the Brownian movement is a natural item for each discerning dealer; however, a few words about the Poisson cycle are all together necessary.

Consider a set of autonomous remarkable irregular variables $\{\tau_i\}_{i\geq 1}$ with boundary λ, that is, with full apportionment. Work $P[\tau_i \geq y] = e^{-\lambda y}$ and let $T_n = \sum_{i=1}^{n} \tau_i$

$$N_T = \sum_{n\geq 1} 1_t \geq T_n. \tag{11}$$

Equation (11) is referred to as having a boundary of λ and being a Poisson process. The total number of transports shown up to time t is a Poisson interaction, for instance, if the holding up events between transports at a bus stop are significantly scattered. A Poisson measure has piecewise steady bearings (right continuous with as much as possible, or RCLL), or bounces of size 1. The gaps (the holding up events) between the leaps, which happen irregularly T_i, are dramatically appropriated. With boundary $t\lambda$, N_t has the Poisson transport at every date $t > 0$; that is, it is a whole number valued and

$$P[Nt = n] = e^{t\lambda}(\lambda t)^n / n!,\ n = 0,1,2,\cdots \tag{12}$$

Equation (12) is the compound Poisson process with $E[N_t] = \lambda t$, where the parameter λ is the intensity function. The Poisson collaboration gives the Brownian development the essential property of stationary and freedom from augmentations; that is, the addition N_t N_s for each $t > s$ has a comparative law as N_t s and is independent of the association's true background up to time s. The cycle featuring independent and fixed additions is referred to as the "toll interaction," named for the well-known French mathematician Paul Levy (Tankov, 2007).

3. Data Analysis and Discussion

3.1. Description of the Data in Terms of Returns

The return for the USD, GBP, EUR, ETB, KES, and TZS against the Rwandan franc is shown in the current figure.

Figure 1 shows that the pound sterling (GBP) is not steady and tends to bunch over time, with a noticeable huge (little) peak or drop in return over time. The euro (EUR) shows an up and downtrend, with a high return from 2015 to 2016 accompanied by high bunch periods. The US dollar (USD) exhibits low

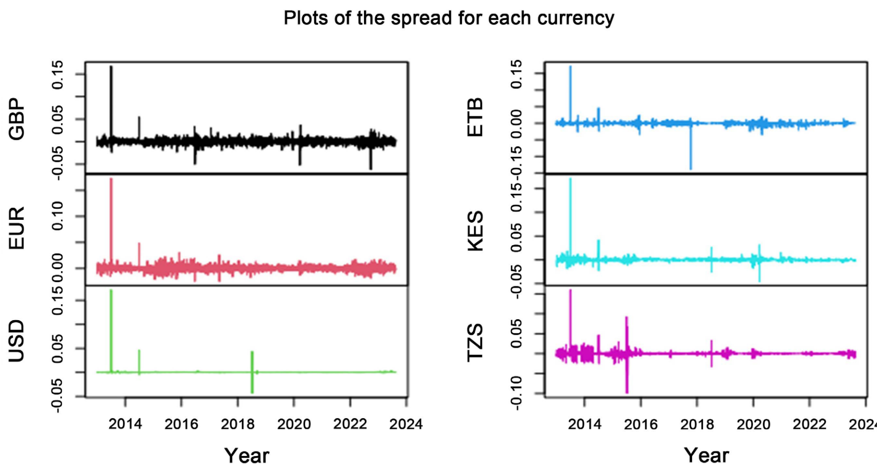

Figure 1. Shows the spread return for every currency.

time-series behavior similar to that of other currencies. Among regional currencies, the Ethiopian Bir (ETB) shows decreasing clustering and is not constant as it tends to cluster over time. The Kenyan shillings (KES) shows cluster from 2013 to 2016 indicating a panic sell-off. From 2016 to 2019, there is no cluster and no trend.

Returns are frequently utilized because they facilitate the normalization of data and facilitate the comparison of assets with varying absolute values.

Comprehending the distribution and patterns of returns becomes essential while executing currency models or fitting financial models. Certain statistical characteristics of returns, like normality or stationarity, are frequently assumed by models. You give the model the information it needs to forecast or estimate parameters based on these assumptions by characterizing the data in terms of returns.

The relationship basically comes down to how the models being used for financial or currency research and the assumptions made about them are met by the data representation that has been selected, such as returns.

3.2. Results (From Fitting the Models)

3.2.1. Running the Model for International Currency EUR, GBP and USD

The compound Poisson model with Gaussian jumps (Norm) fit, empirical density (continuous line) against the geometric Brownian motion (gBm) fit, and Normal Inverse Gaussian (NIG) model density are shown in the current figures below.

The densities of the assumed gBm (horizontal dashed green line), Norm (dashed red line), and NIG (blue dashed line) densities are plotted against the density of the experimental data H_t (solid line) in Figure 2, Figure 3 and Figure 4. The densities of the assumed gBm, Norm, and NIG were approximated using the Poisson measure of the degenerate mixture in Equation (10). In Figure 2, NIG is blue, which aims to balance the density of observational data; red (Norm) is next, and gBm comes last. In Figure 3, the NIG, which is blue, is the highest,

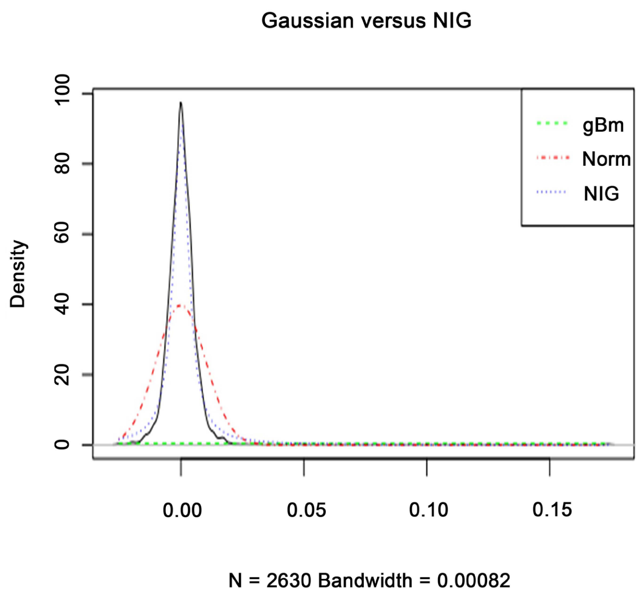

Figure 2. Depict the three models that were used to fit the euro (EUR).

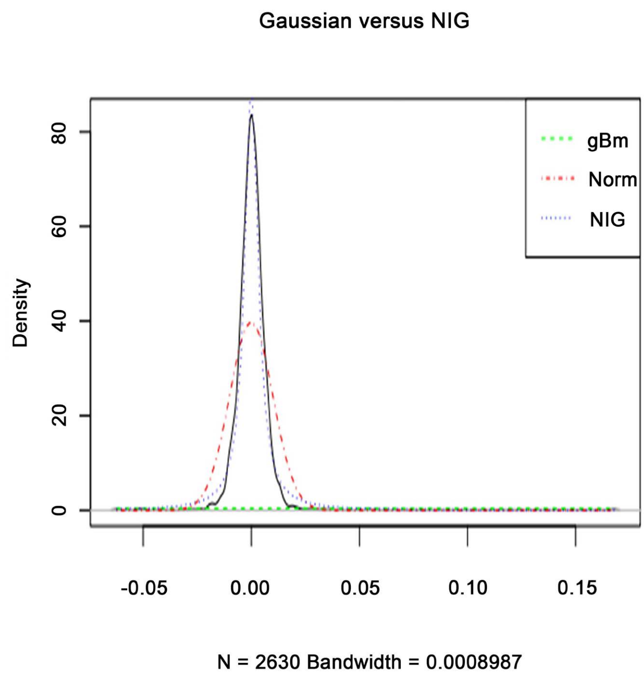

Figure 3. Depict three models that were fitted with British Pound sterling (GBP).

followed by the empirical density, then the norm, which is red, and gBm (green) come last. In **Figure 4**, the empirical density is the highest, followed by NIG, though all three densities seem to deviate from the empirical data set. The best densities are the blue NIG in all the figures above, which is followed by the densities of the Norm (red); gBm, on the other hand, completely deviates from the density of observational data; suffice to say that gBm is the worst case in **Figure 2**, **Figure 3** and **Figure 4**. In this specific data set, the data is obviously not

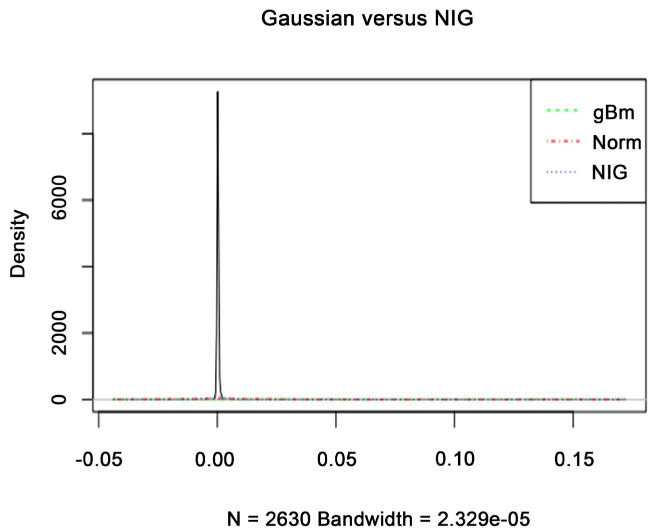

Figure 4. Displays the three models that the US dollar (USD) was fitted with).

Gaussian, and, in particular, it does not originate from gBm, but rather it can have a complex Poisson type with an NIG. This empirical evidence is also confirmed by evaluating the Akaike information criteria using the AIC function.

3.2.2. Running the Model for Regional Currency ETB, KES and TSZ

The empirical density (continuous line) is plotted against the geometric Brownian motion (gBm) fit, the compound Poisson model with Gaussian jumps (Norm) fit, and the density of the Normal Inverse Gaussian (NIG) model in the current figures.

The densities of the assumed gBm (horizontal dashed green line), Norm (dashed red line), and NIG (blue dashed line) densities are plotted against the density of the experimental data H_t (solid line) in **Figure 5**, **Figure 6** and **Figure 7**. The densities of the assumed gBm, Norm, and NIG were approximated using the Poisson measure of the degenerate mixture in Equation (10). In **Figures 5-7**, NIG (blue) seeks to balance the empirical data, followed by norm (red) and finally gBm (green). In this specific data set, the data is obviously not Gaussian, and, in particular, it does not originate from gBm, but rather it can have a complex Poisson type with an NIG. The best densities are the blue NIG, which is followed by the densities of the norm (red); gBm, on the other hand, completely deviates from the density of observational data; suffice to say that gBm is the worst case. This empirical evidence is also confirmed by evaluating the Akaike information criteria using the AIC function.

3.3. Validation of the Model

3.3.1. Selection Criteria

The two techniques listed below are for selecting the optimal or most effective model using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

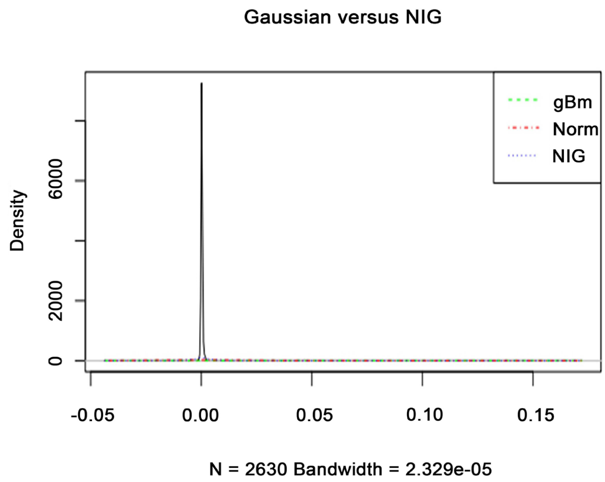

Figure 5. Shows the three models that the Ethiopian Birr (ETB) was fitted to.

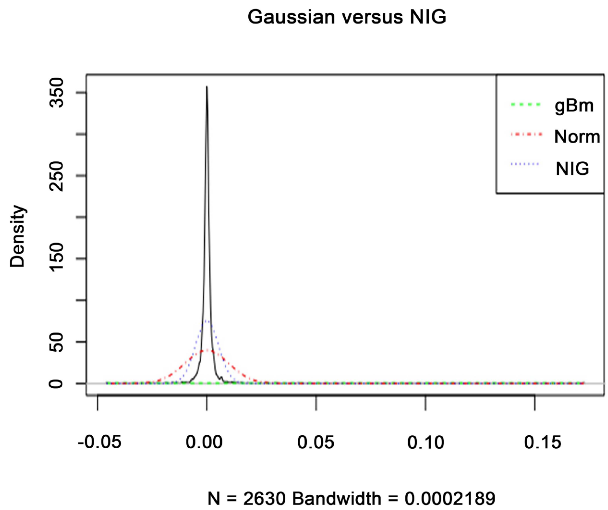

Figure 6. Shows the three types of models that the kenyan shillings was fitted to.

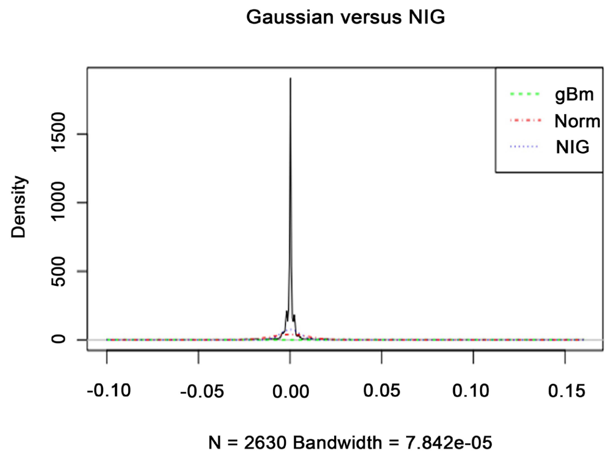

Figure 7. Shows three models that were fitted to Tanzanian Shillings.

1) Akaike Information Criterion (AIC): In order to examine several models for a given result, Akaike initially suggested the AIC. The following is an illustration of the candidate model's AIC:

$$\text{AIC} := -2l\left(y \mid \hat{\theta}\right) + 2J, \tag{15}$$

where $\left(y \mid \hat{\theta}\right)$ is a log-likelihood at the surveyed model's most limit place and J is the number of assessed boundaries in the model, including the catch. The standard for judgment: a model's quality increases with its AIC value.

2) BIC (Bayesian Information Criterion): First introduced by Schwarz, BIC one of the times calls the Bayesian data criterion (BIC) or Schwarz criterion (also SBC, SBIC) which is a basis for model choice among a limited arrangement of models. The following describes the competitor model's BIC:

$$\text{BIC} := -2l\left(y \mid \hat{\theta}\right) + \ln(n) J, \tag{16}$$

where n is essentially the size of the sample; J is the number of evaluated boundaries in the model including the catch and $\left(y \mid \hat{\theta}\right)$ is the log-likelihood at its most limit spot of the surveyed model. The norm of decision: the more unassuming the value of BIC is, the better the model. The methodology for applying AIC and BIC are given as follows:

Step 1: Selecting up-and-comer models which can be fitted to the dataset. Step 2: Estimating unknown parameters of models. Step 3: Finding upsides of AIC and BIC by utilizing the formulas (15) and (16), respectively. Step4: Basing on the standard of choice, one can choose the most reasonable model (Pho, Ly, Ly, & Lukusa, 2019).

3.3.2. Comparison Performances of Models

Models will be validated in this section based on their AIC and BIC scores.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in **Table 1**. Additionally, the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in **Table 2**. Additionally, the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in **Table 3**. Additionally, the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in **Table 4**. Additionally, the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in **Table 5**. Additionally,

Table 1. Give each model's BIC and AIC scores for EUR.

	NIG	Norm	gBm
BIC	−18296.86	−17284.92	2exp+10
AIC	−18320.35	−17296.67	2exp+10

Table 2. Give each model's BIC and AIC scores for GBP.

	NIG	Norm	gBm
BIC	−18296.86	−17292.15	2exp+10
AIC	−18320.35	−17303.9	2exp+10

Table 3. Give each model's BIC and AIC scores for USD.

	NIG	Norm	gBm
BIC	−18527.59	−15479.26	2exp+10
AIC	−18551.09	−15491.01	2exp+10

Table 4. Give each model's BIC and AIC scores for ETB.

	NIG	Norm	gBm
BIC	−20042.86	−18369.91	2exp+10
AIC	−20066.36	−18381.66	2exp+10

the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

A lower Akaike information criterion (AIC) is found for the compound Poisson with Normal Inverse Gaussian jumps (NIG) model in Table 6. Additionally, the Bayesian criteria information (BIC) of the model NIG is smaller. Therefore, when compared to the model gBm and Norm, the model NIG is most likely.

3.4. Summary of Model's Performance

After running the model for the euro, British Pound Sterling, United States Dollar, Ethiopian Birr, Kenyan Shillings, and Tanzanian Shillings, the model compound Poisson with normal inverse gaussian jumps (NIG) has the lowest value for both the Akaike information criterion and the Bayesian information criterion, followed by the model compound Poisson with gaussian jumps and then geometric Brownian motion. In contrast to the compound Poisson model with gaussian jumps and the geometric Brownian model, the model with normal inverse gaussian jumps has fit the Rwandan forex market return quite well.

4. Conclusion and Discussion

4.1. Conclusion

The study's primary goal was to assess the Levy process on the returns from the Rwandan FX market utilizing 2631 observations from 02/01/2013 to 18/08/2023. The study specifically sought to match the returns from the Rwandan FX market using the gBm, Norm, and NIG models, and it evaluated each model's

Table 5. Give each model's BIC and AIC scores for KES.

	NIG	Norm	gBm
BIC	−20042.86	−18715.11	2exp+10
AIC	−20066.36	−18726.86	2exp+10

Table 6. Give each model's BIC and AIC scores for TZS.

	NIG	Norm	gBm
BIC	−20066.36	−17106.82	2exp+10
AIC	−20066.36	−17118.57	2exp+10

performance using the AIC and BIC metrics. The research determined that, after fitting the three models for each currency, the model Compound Poisson having Normal Inverse Gaussian jumps (NIG) with AIC = −18320.35 and BIC = −182926.86 is the best model for the euro (EUR) in comparison to the models' geometric Brownian motion (gBm) with AIC = 2exp+10 and BIC = 2exp+10 and Compound Poisson having Gaussian jumps (Norm) with AIC = −17296.67 and BIC = −17284.92. The study also discovered that, in terms of the model's geometric Brownian motion (gBm) with AIC = 2exp+10 and BIC = 2exp+10, and Compound Poisson having Gaussian jumps (Norm) with AIC = −17303.9 and BIC = −17292.15, the model Compound Poisson having Normal Inverse Gaussian jumps (NIG) with AIC = −18320.35 and BIC = −18296.86 is the best model for Pound sterling (GBP). The study also discovered that the model Compound Poisson having Gaussian jumps (Norm) with AIC = −15491.01 performs marginally better than the model Compound Normal Inverse Gaussian jumps (NIG) with AIC = −18551.09 and BIC = −18527.59 for the US currency (USD). In addition, the study discovered that the model Compound Poisson having Normal Inverse Gaussian jumps (NIG) with AIC = −20066.36 and BIC = −20042.86 is the most effective model for Ethiopian Birr (ETB) when compared to the model's geometric.

Brownian motion (gBm) with AIC = 2exp+10 and BIC = 2exp+10 and Compound Poisson having Gaussian jumps (Norm) with AIC = −15491.01 and BIC = −15479.26. Additionally, the model Compound Poisson with Normal Inverse Gaussian jumps (NIG) with AIC = −20066.36 and BIC = −20042.86 outperforms the model Compound Poisson with Gaussian jumps (Norm) with AIC = −18726.86 and BIC = −18715.11 for Kenyan shillings (KES), according to the study. A comparison between the model geometric Brownian motion (gBm) with AIC = 2exp+10 and BIC = 2exp+10 and the model Compound Poisson having Gaussian jumps (Norm) with AIC = −17118.57 and BIC = −17106.82 reveals that the model with the best performance for Tanzanian shillings (TZS) is the one with Normal Inverse Gaussian jumps (NIG) with AIC = −20066.36 and BIC = −20066.36.

4.2. Recommendation

The study's empirical findings indicate that, across all currencies, the NIG model performs better than the Norm and gBm models. The Nairobi Securities Exchange (NSE), Dares Salaam Stock Exchange (DSE), Rwanda Stock Exchange (RSE), and Lusaka Stock Exchange (LuSE) could all adopt the model NIG.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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