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Central Limit Theorem to Approximate Aggregate Risk of Portfolio: Using the ModelRisk Software

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this note, the non-sampling information in portfolio management is considered. These information may be the past belief of investor about a special asset. They are characterized as the correlated binary random variables. Then, the Monte Carlo is applied to derive the posterior distribution of binary variables given the past returns which indicates the tendency of investor to keep or drop a portfolio via using the non-sampling and sampling information simultaneously. The posterior distribution of belief of investor and the accuracy of Bayesian method are shown via plotting histograms.

Keywords: Copula; dirichlet distribution; mixture distribution; ModelRisk software.

1. INTRODUCTION

Modern portfolio management (MPT) is highly related to theoretical contributions of [1]. The MPT has been extended substantially by the Capital Asset Pricing Model (CAPM) of [2]. However, in portfolio management, to reduce the unsystematic risk, a diversified different financial assets is held in a portfolio. However, from a Bayesian point of view to the problem, investors have prior information about including or excluding a special asset in a specified portfolio. These are important non-sampling information which should be considered in determining weights of assets. These information may be the past belief of investor about a special asset.

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Therefore, let $(J_1, \dots, J_{n-1})$ is a vector of Binary random variables comes from the Dirichlet distribution. Here, J_i is one if $i - \text{th asset exists in portfolio}$ and zero, otherwise. Let $J_n = n - \sum_{i=1}^{n-1} J_i$.

Let R_i be the return of i-th asset. If an asset exists in the portfolio its share is π_i. Then, the portfolio return is given by

$$R_p = (1/n) \sum_{i=1}^{n} J_i \pi_i R_i,$$

such that $(1/n) \sum_{i=1}^{n} J_i \pi_i = 1$. Obviously, the distribution of R_p is mixture, which its exact functional form is too difficult. Let $R_i^* = \pi_i R_i$.

By implementing the Markovitz mean-variance portfolio selection on R_i^*'s, weights π_i are derived by minimizing the $var(R_p)$, for a predetermined level of $E(R_p)$. Thus, in this paper, it is assumed that π_i's are known. A useful instrument in this problem is the posterior distribution of $(J_1, \dots, J_{n-1})$ given observing the historical data R_p. More precisely, what the posterior probability of configuration of $(j_{i_1}, \dots, j_{i_{n-1}})$ is?, i.e.,

$$P(J_1 = j_{i_1}, \dots, J_{n-1} = j_{i_{n-1}} | R_p).$$

Studied the normal approximation and normal power (NP) approximation for mixture distribution and concluded that NP approximation is too useful [3]. They used the Monte Carlo simulation to obtain the accuracy of approximation. ModelRisk (see [4]) is an Excel-based software for Monte Carlo simulation, sensitivity analysis and optimization in financial problems. It has many applications in advanced risk analysis. ModeRisk is applied by researchers in financial literatures. For example, [5] introduced the concept of Net present value at risk (NPVaR) for financed infrastructure projects [6]. applied the Monte Carlo method for risk analysis for large engineering projects by modeling cost uncertainty for ship production activities [7]. Applied Monte Carlo simulation of investment returns for toll projects [8]. studied the business valuation under uncertainty (considering dependency between cash flows) using copula concept [9]. considered sensitivity assessment modeling in European funded projects proposed by Romanian companies.

The rest of paper is organized as follows. In this paper, to find the marginal distribution of R_p and posterior $(J_1, \dots, J_{n-1})$ given R_p, the Monte Carlo is used. First, returns of each asset are assumed to be independent, mutually, however, several types of correlations based on copula function is considered in the simulation section.

2. SIMULATIONS

In this section, two examples are considered. The first example studies the central limit theorem approximation for aggregate risk of a portfolio. The second example considers the effect of correlation between assets and the use of copula function in modeling the correlation.

Example 1. The data is taken from Excel sheet entitled "CLT_risk_portfolio_approximation" downloadable from "Example models and tutorial videos - Vose Software[1]". The portfolio contains 132 risky assets with Lognormal, PERT, Normal, ModPERT and Triangle distributions. Investor belief on each risk assets comes from independent Bernoulli random variable. The following histogram (Fig. 1) shows the marginal distribution of portfolio return, $\mathbf{R_p}$. The number of simulation is 1000.

Some summaries statistics are given as follows Table 1.

The value at risk (VaR) for some significance level α are given as follows (V_0 is the initial value of portfolio).

However, if the belief of investor isn't included to the problem, then the density of , $\mathrm{R_p}$ is as follows (Fig. 2). Again, some summaries statistics are given as follows. In this special case, the performance of Markovitz portfolio and the $\mathrm{R_p}$ are the same.

Example 2. In this example, the Gaussian copula is considered between all financial assets. The following histogram (Fig. 3) shows the probability density function of $\mathrm{R_p}$.

It is seen that the use of copula improves the skewness and kurtosis of marginal distribution of portfolio return.

[1]http://www.vosesoftware.com/vosesoftware/ModelRiskHelp/Resources/Example_models/Example_models_introduction.htm

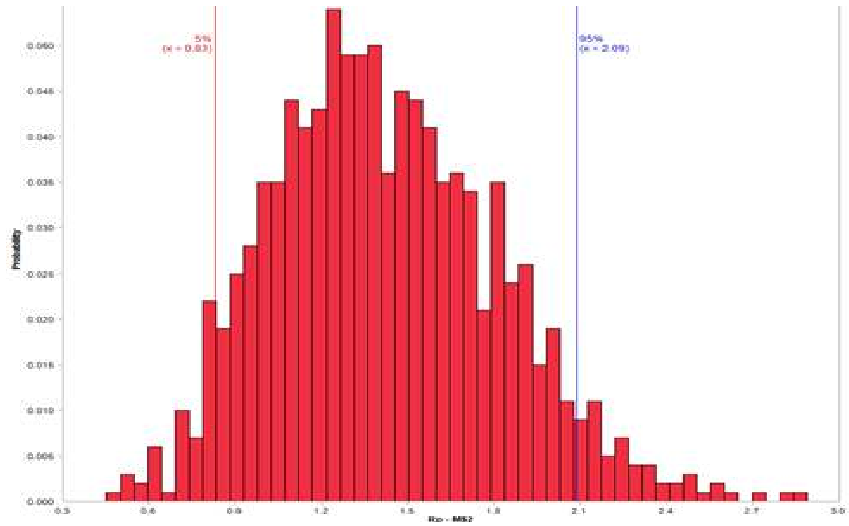

Fig. 1. Marginal density of R_p

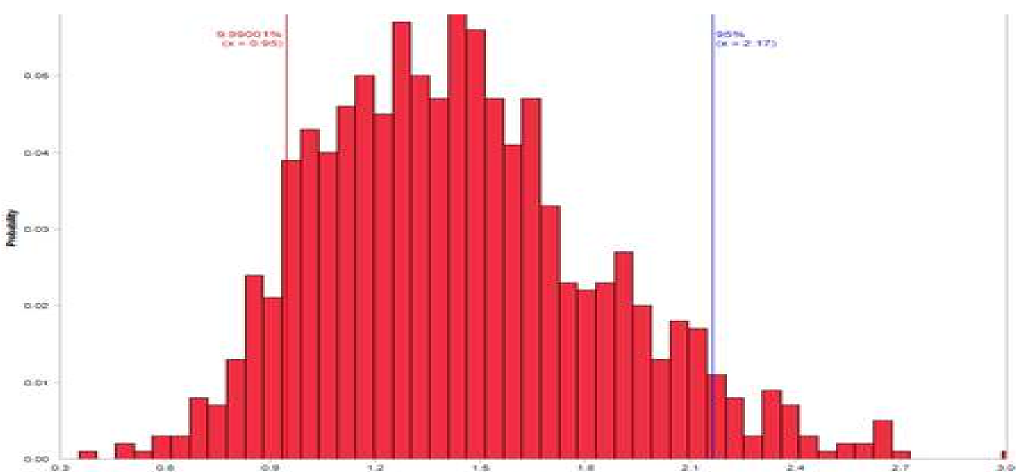

Fig. 2. Density of usual R_p

Table 1. Summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.45	1.42	2.84	0.29	0.153	0.275	0.406	3.04

Table 2. VaR for some significance level α

α	0.005	0.01	0.05	0.1
VaR	$0.58V_0$	$0.62V_0$	$0.83V_0$	$0.94V_0$

Table 3. Summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.35	1.43	2.99	0.404	0.163	0.28	0.5	3.15

Table 4. Summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.35	1.43	2.99	0.294	0.087	0.206	0.9	4.1

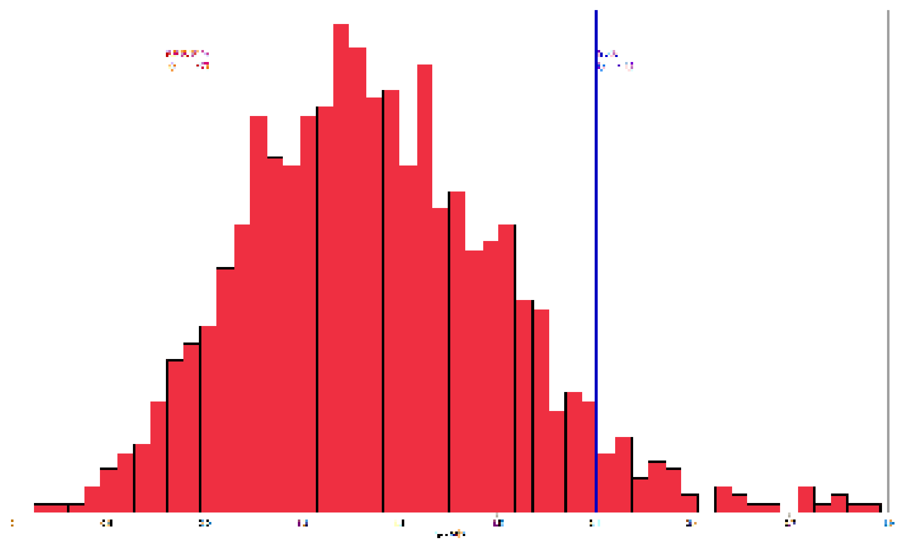

Fig. 3. Marginal density of R_p

3. TWO CONCEPTS

In this section, two important concepts are studied. The first is the posterior distribution of investor belief and the second is the stochastic programming applied in this paper.

3.1 Posterior Distribution

An MCMC method is really suitable for estimating the posterior distribution of the investor belief sequence. The reversible jump algorithm is based on the fact that the dimension of the model can change, according to the number of segments. Unfortunately, this algorithm converges slowly, and many iterations are needed for estimating correctly the posterior distribution. Therefore, the Hastings Metropolis algorithm with the Gibbs sampler is applied. The following Table gives some posterior values for investor belief for Example 1. In the Table, for example, (95,125) means the investor has selected the assets numbered 95 and 125.

3.2 Stochastic Programming

Consider the portfolio management in which the return of portfolio by considering the pre-information of investor is maximized as well as the diversified risk should be in controlled. That is, to

$$\max\,(E(\mathrm{R_p}))$$

s.t. $var(\mathrm{R_p}) \leq \mathrm{LB}$. The following Table gives some values of weights for the first asset by this method.

Table 5. Posterior belief of investor

(1,2,3)	(3,4,75)	(60,61,62)	(95,125)	(1,…,132)	(1,…,66)	(67,…,132)	(131,132)
0.09	0.1	0.05	0.09	0.1	0.51	0.60	0.55

Table 6. The weight of the first asset

LB	0.05	0.1	0.15	0.2	0.5	0.75	0.9
Weight	0.023	0.051	0.055	0.063	0.064	0.081	0.1

4. CONCLUSIONS

The portfolio management is an important part of finance. There are many well-known techniques such as efficient frontier method to find the best combinations of assets in a portfolio. In the regular methods, only the statistical properties of returns of assets are considered. However, sometimes, investors have belief about the returns. This fact motivates the use of Bayesian methods in portfolio management. This paper considers this concept and uses the computational tools such as MCMC and stochastic programming methods to perform the portfolio management using posterior belief of investor distribution. This paper also uses the ModelRisk software.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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