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Sierpinski Carpet and Chaos in the Periodic Table of the Elements

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Fractals are self-similar geometric pattern which can be found in nature. They have applications in mathematic, electronic, architecture. Fractal sets also can be used to create chaotic systems. This work is about applying Sierpinski carpet order on the periodic table of the elements to create a new pattern for the chemical elements. Fibonacci numbers and Math lab software are used to transform a linear system to three spiral systems. This new pattern which is consisted of three layers shows that the flows among chemical elements are based on Archimedes spiral equation The purpose of this study is to show Sierpinski carpet order in the periodic table of the chemical elements and also there can be a chaos even in chemical elements.

Keywords: Sierpinski carpet; Fibonacci numbers; golden spiral; the periodic table of the elements; Archimedes spiral equation; Chaos.

1. INTRODUCTION

Fractals can be described as series of geometries which make patterns similar to the whole [1]. The self-similarity is an important character of Fractal sets and each part of the fractal must has the same or near shape of the whole fractal [2].There are famous and known

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fractals such as Cantor set, Julia set, Sierpinski triangle and carpet, Koch curve and Mandelbrot set which are considered as subjects of interests in many researches. The most famous Sierpinski type fractals which are Sierpinski triangle, Sierpinski gasket and Sierpinski carpet have been center of many studies and attentions, recently[1].

Fractal structure can be lead to create a chaotic system but creating fractals alone is not sufficient enough to create chaos and all fractals can not be considered as chaotic [3]. There are different types of chaos with one, two and three dimensions such as RÖssler chaos and Shilnikov chaos [4,5].

One of most famous fractal set is Sierpinski carpet which is a two dimensional fractal . The first step to drew it, one must divide a full or empty square into nine sub-squares and then remove the middle one. The result of this step is eight sub-squares remaining. For the next step, this procedure will be applied to all remaining squares. The Repetition of this process for a number of times on the remaining squares leads to the (f) step in Fig (1) which finally Sierpinski carpet will be created [6]. The resulted figures in each step of generating Sierpinski carpet are shown in Fig.1.

In19th century chemists knew several elements and their chemical and physical properties, so a systematically arrangement of elements became an important task. By using 'Law of Octaves' which was declared by John Newlands, in 1869, Dmitri Mendeleev arranged chemical elements in order of their atomic weights and created the table with groups and rows. There were eight basic groups for some of elements. The remaining elements was placed between second and third groups. Some of the places in the periodic table was empty and Mendeleev considered them as elements which will be discovered, later [8].

About 1923, Horace Deming proposed a modification on the Mendeleev's periodic table. As he showed the periodic table with 18-columns consisted of all the elements which have been discovered before and after Mendeleev's time. This change became the basic of the present periodic table of the elements and today this table have 118 elements in 18 groups which are shown in Fig. 2. [8].

Most researches are about creating order in a chaotic system, but this paper is an original research for using Sierpinski carpet set order to create a new pattern for the periodic table of the chemical elements and lead the pattern to a chaos. The Archimedes spiral equations in the Sierpinski carpet version of the periodic table is discussed to prove of a chaos.

2. COMPUTATIONAL METHOD

This method provides the graphical analysis of the behavior of atomic number of elements and Fibonacci numbers after elements are put in sierpinski carpet order , so MATLAB software is used for drawing the resulted spiral.

3. RESULTS AND DISCUSSION

3.1 Sierpinski Carpet and Elements of and p Block

The periodic table of elements consists of 18 groups and 7 periods (Fig. 2). Groups1 and 2 are called s block due to filling Their s orbitals. Groups 13-18 are called p blocks because their p orbital are filling[9]. There are 50 elements in s and p block which are going to be organized as a Sierpinski carpet fractals.

To create a suitable Sierpinski carpet by using Fig. 1) and a stage for building a Sierpinski carpet, a main square is selected and divided to nine equal squares. As it was described, the middle square must be removed and eight squares are remaining. In this case, Hydrogen is placed as the middle square which means that it is an element which belong to no group but stands on top of the first group alone. The eight sub-squares resulted of the first step are given to elements Helium, Lithium, Beryllium, Boron, Carbon, Nitrogen, Oxygen and Fluorine. These elements are the representing and the first elements of the groups 1,2, 13,14,15,16,17 and 18 [9].

Fig. (3a) is the resulted stage of Fig.1) or (1f) [7] which shows that by creating a typical Sierpinski carpet 512 squares will be obtained, but there are 50 elements in s and p blocks. Using Fig (3a) as a model, For replacing elements in Sierpinski order, some of squares should be omitted. Fig (3b) shows in the first circle, one square is omitted of each two squares. Also, in the second circle in Fig (3b) two squares is omitted of each three squares. Finally, all squares in the third row of figure (3d) is omitted except one of them. In Fig (3e) the remained squares are divided to smaller squares. This way the pattern for replacing elements of the s and p blocks is prepared. This kind of changes and omitting method is done for the first time on a typical Sierpinski carpet 512 squares by the author to create a platform for theses elements. The stages are shown in Fig (3).

The elements of s and p block are used To fill the squares of the created platform . Therefore, the first nine elements are arranged in the middle main square. Elements with atomic number 10 to 17 are placed as the second round of squares. This round of squares indicates that the p block elements in these squares have empty d orbitals. The eight bigger squares which each are divided to four sub-squares are for the remained elements of each group which have full d orbitals and their p orbitals are filling. On the top of the carpet there is element 118 which shows the end of seventh period. The arranged s and p block elements as Sierpinski carpet are shown in Fig (4).

3.2 Sierpinski Carpet and Elements of d and f Block

In the periodic table, There are elements which are arranged in groups 3 to 12 and period 4 to7 (Fig.1). The main character of the elements in those four rows in the periodic table is that from left to right their d orbitals are filling. The ones in these four rows are called transition metals or d block elements. In the fourth and fifth periods of the periodic table (first and second rows), there are ten elements (21-30 and 39-48 atomic numbers) which are continued by p block elements (groups 13-18) .There are 14 elements in the third and the fourth rows (the sixth and seventh period) which are called Lanthanides and Actinides. Lanthanides contain elements with atomic numbers 57-80 and Actinides are the elements with 89-112 atomic numbers. These elements' d orbitals will be filling after their f orbitals are filled. Thus the Sierpinski carpet order and the chaos for the d block elements of periods 4 and 5 will be different from 6 and 7 periods.

3.2.1 Transition metals of 4 and 5 periods

There are 10 elements in each of the first and the second rows which contain elements with 21-30 and 39-48 atomic numbers [9]. Using Fig (3e) of Fig (3) and choosing only the middle square and a single one outside, Fig (5a) will be resulted. The Sierpinski carpet order is a main square divided to nine sub-square and one single square which are arranged for each row as it is shown in Fig (5a).

The first elements of first two rows which have d^1 orbital are put in the one single square's place outside and the nine other elements are put in the place of the nine divided sub-squares. The replacement order for the elements of the two rows are shown in Fig (5b) and it shows that the resulted fractals are mirror images.

3.2.2 Transition metals of 6 and 7 periods

The patterns for elements of 6 and 7 periods (Lanthanides and Actinides) which both their d and f orbitals are filling, is different and changed. In each row, first there are 14 elements with f orbitals and after that there are 10 elements which their d orbitals began to fill [9].

Fig (6) shows thst Lanthanum (atomic number 57), Actinide (atomic number 89) both with d^1 orbital, Cerium (atomic number 57) and Thorium (atomic number 90) both with f^1 orbital are arranged as four squares of nine divided sub-squares in the central main square of the platform (Fig 6).

Following Lanthanum and Actinide, the other d block elements of each two rows are arranged in the place of nine sub-squares but the order for other f block elements Followed by_Cerium and Thorium is a square divided to nine sub- squares and four single squares around the main one. The resulted order and pattern is shown in Fig (6). In this pattern, Lanthanum which is in the central square is not only followed by Cerium of f block series in one direction ,but also followed by d block elements in another direction. This patten has happened for Actinide group elements, too.

3.3 Sierpinski Carpet Chemical Elements and Self-Repeatability

Fig (7) is the combination of Figs (4), (5) and(6) which makes the periodic table of the elements a three layer pattern Sierpinski carpet fractal. Each layer has a characteristic pattern and self-similarity in each layer is quiet obvious.

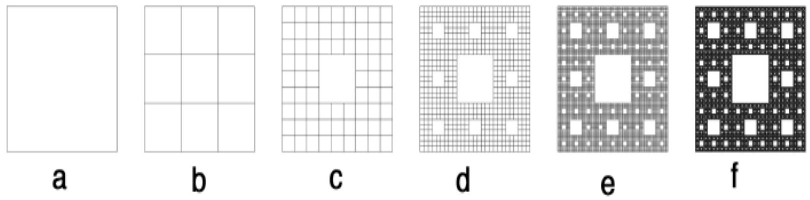


Fig. 1. Steps in generating Sierpinski carpet [7]

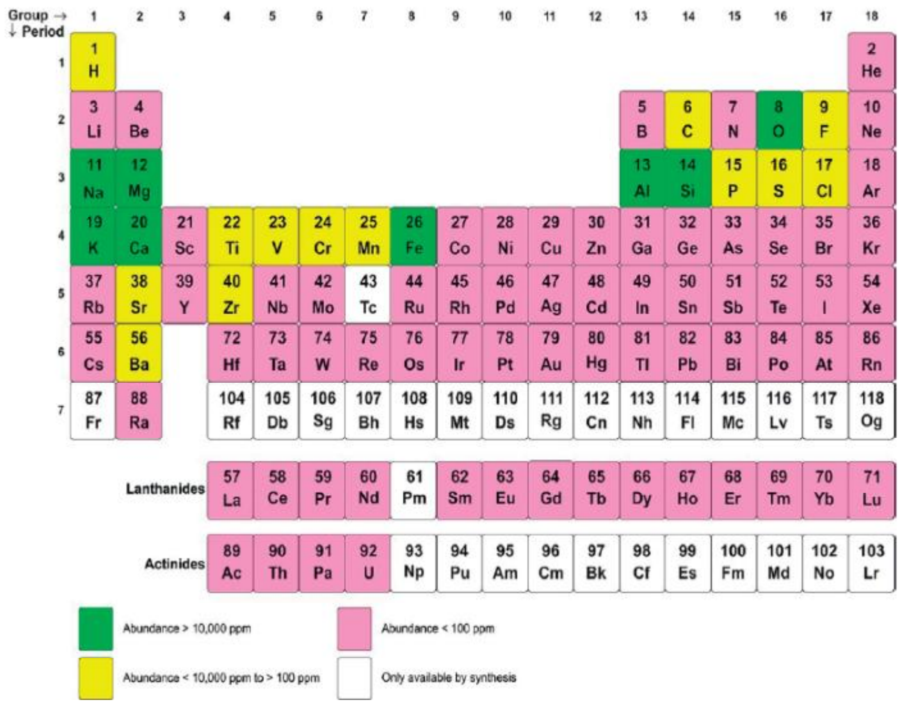


Fig. 2. Periodic tables of elements and their abundance in the Earth's crust[9]

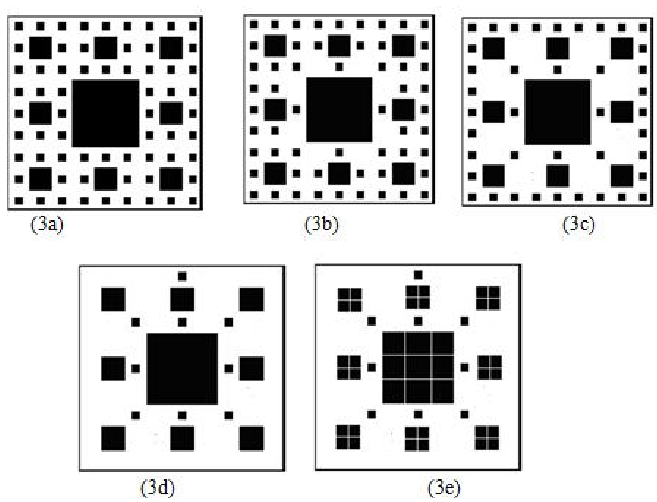


Fig. 3. Stages of omitting squares in original Sierpinski carpet

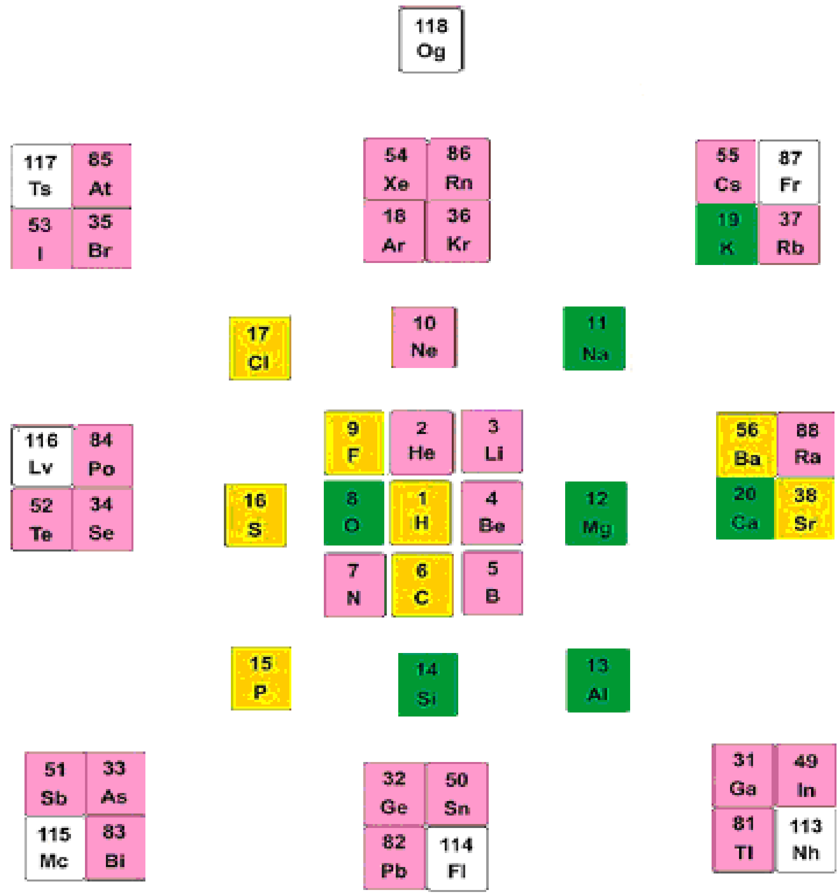
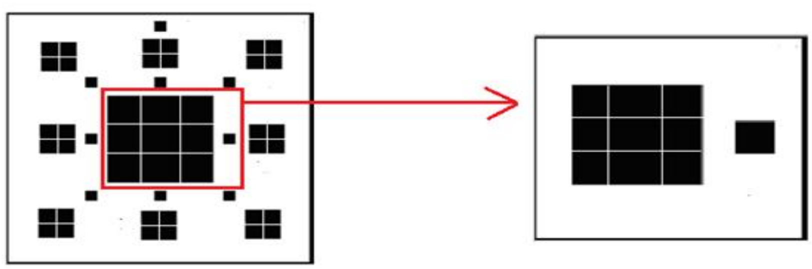
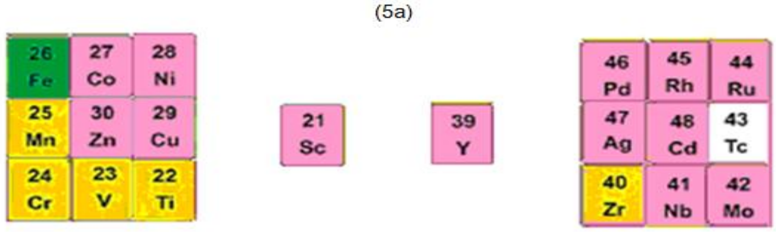


Fig. 4. elements of s and p block in Sierpinski carpet order



(5a)



(5b)

Fig. 5. (a) The chosen fractal for elements period 4 and 5(b) for elements period 4 and 5 in Sierpinski carpet order

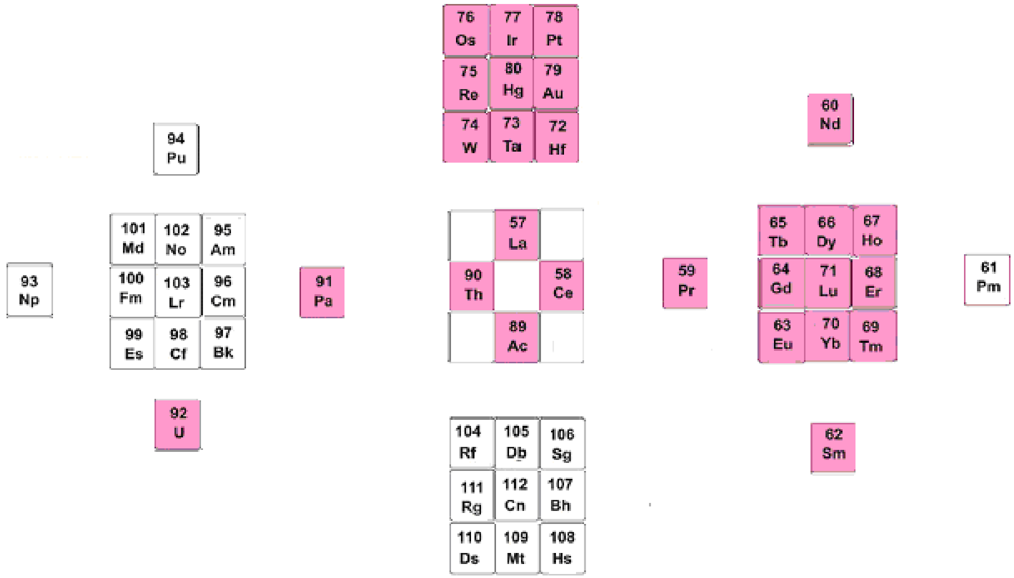


Fig. 6. elements period 6 and 7 in Sierpinski carpet order

If new elements of the eighth period discover, considering Fig (4), the assumed order for the new s and p block elements can be shown as Fig (8). Fig (8) shows that the elements of the eighth period are placed as single squares in the Sierpinski carpet order. This figure also indicates that the elements of next periods will be set in the place of four divided sub-squares after the single squares are completed in the pattern.

It means that there is a pattern of self-repeatability in the elements of 18 groups when they are arranged as Sierpinski carpet fractal. So the first layer of Fig (7) is repeatable.

However self- repeatability property will not happen for the transition elements of 4 and 5 periods (21-30 and 39-48 atomic numbers) which only have d orbitals to be filled, because new discover elements will not join these periods. Thus the second layer in Fig (7) remains unchanged and unrepeated.

Fig (9) indicates that new discovered elements of the eighth period which will have d and f orbitals to be filled (elements 121-144) can show self-repeatability in their Sierpinski carpet pattern But this self-repeatability will not be like completing the main pattern as it was with the first layer (s and p block), but it will be organized and placed as a new layer after the third layer and will make a fourth layer . The new Sierpinski carpet order of elements along with the eighth period elements is shown in Fig (10).

Fig (10) shows The Sierpinski carpet fractals and order when new elements of future eighth period are discovered. In this figure self-repeatability to complete the layer of elements of s and p block and an additional new layer for new d and f block elements can be seen.

3.4 Fibonacci Numbers and Golden Spiral

The Fibonacci numbers are the sequence of numbers such as 0,1,1,2,3,5,8,13, 21,34,55,89,144, where $F_n=F_{n-1}+F_{n-2}$ which means each number in the sequence is formed by adding the previous two. As the sequence shows the initial values for F_0 and F_1 are 0 and 1, respectively [10]. Equation (1) indicates a approximating formula to calculate the nth Fibonacci number. Also equation (2) shows the formula for the ratio $\frac{F_{n+1}}{F_n}$ which also called as Golden *Ratio* (φ) and its value is approximately 1.6180339[11].

$$F_n \approx \frac{1}{\sqrt{5}}[\frac{1+\sqrt{5}}{2}]^n \tag{1}$$

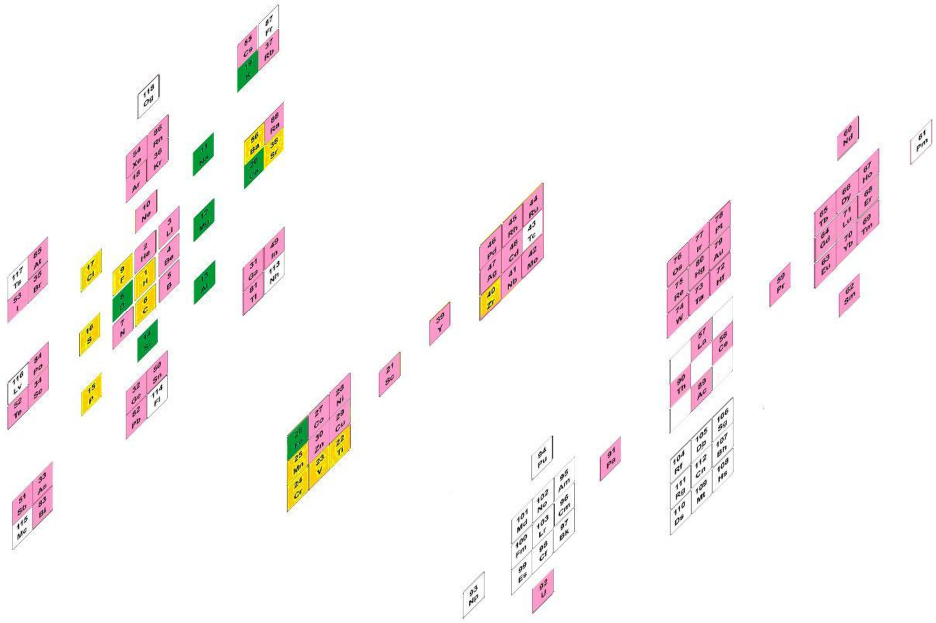


Fig. 7. Three layer of periodic tables based on Sierpinski carpet

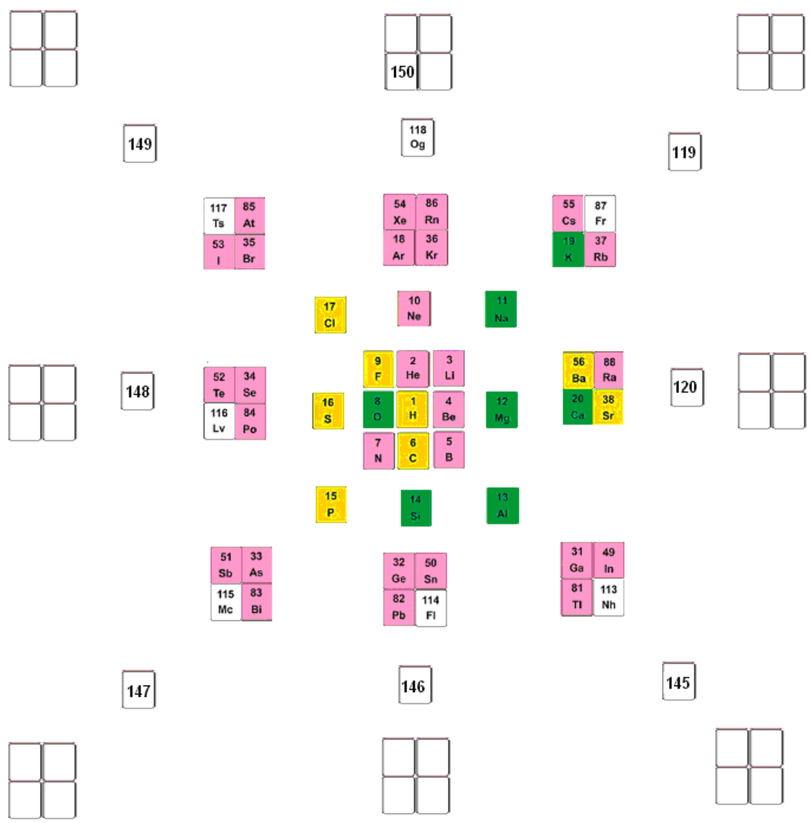


Fig. 8. Self-repeatability of s and p bolck

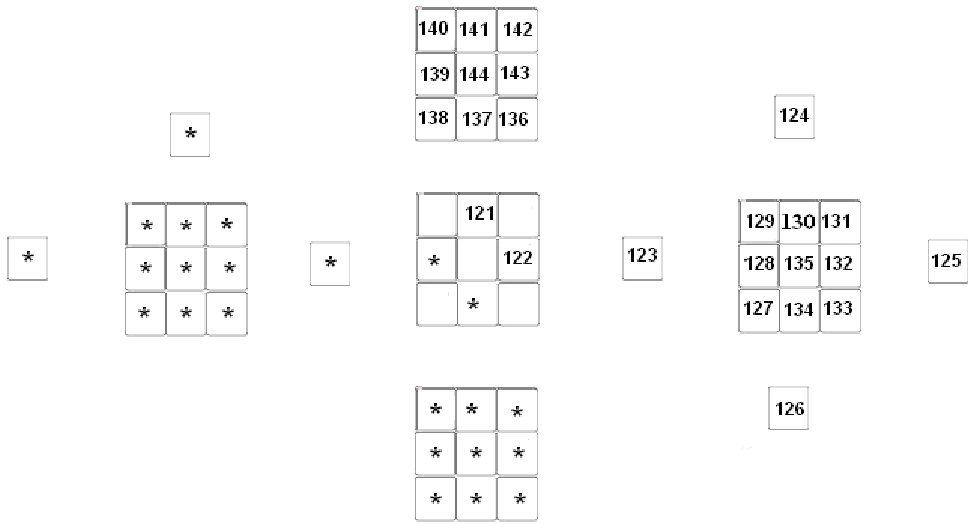


Fig. 9. The fouth layer of elements

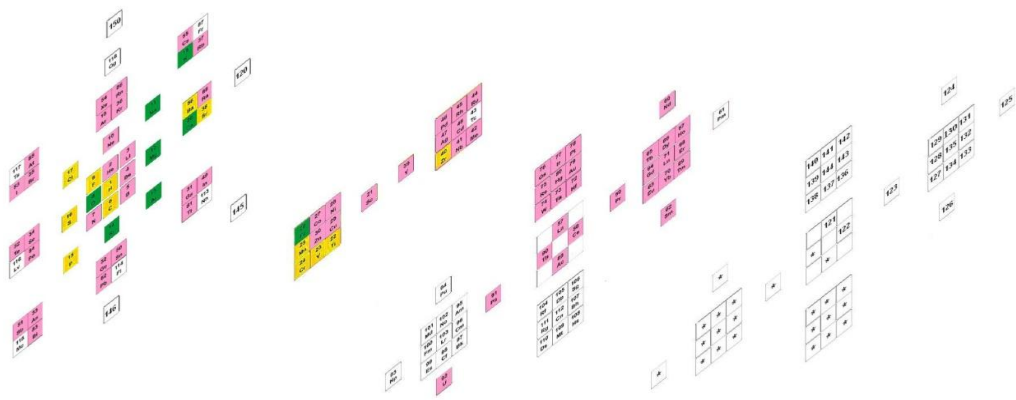


Fig. 10. Sierpinski carpet order for all the eight periods of chemical elements

$$\frac{F_{n+1}}{F_n} \approx \frac{1+\sqrt{5}}{2} \tag{2}$$

Normally no curve of golden ration can be drawn in the periodic table of the elements. However when the elements are arranged as Sierpinski carpet , a curve can be drawn. In the Fig (4) (s and p block) there are elements with atomic numbers 1,2,3,5,8,13, 34,55 which are same as Fibonacci sequence but in this new version, numbers 0, 21, and 89 are missing. To investigate that the pattern of s and p block elements order by Sierpinski carpet can create a golden ratio even without three of Fibonacci numbers, the new sequence of 1,2,3,5,8,13, 34,55 is drawn with help of Matlab software. The curve must drawn in a way that passes the palace through or near that number in Sierpinski carpet. To draw the best curve as possible, $\sqrt{5}$ in equation (2) must change to $\sqrt{4.44}$. The new version of golden ratio is shown in Fig (11) with value φ approximately 1.5536. This result can indicates that the resulted Sierpinski carpet fractal of elements can be a nonlinear system and perhaps a chaos theory can be define for it.

3.5 The Chaos System

3.5.1 The resulted orbits and curves

The main purpose is to create a spiral chaos in the three layers shown in Fig (7). To achieve the chaos, the elements must create spiral systems, so all the chemical elements in each layer will be connected with same center circles. Then, those circles in each layer will be connected in a way to make a characteristic and unique curve or curves for that layer. The circles, the flows and the curves for each layer are shown in Figs (12), (13) and (14).

Fig (12) shows one common curve and one main flow for the layer which is consists of s and p block elements . The equation of the curve is based on Archimedes spiral and it is $r = a(\pi - \theta)$.

Fig (13) shows the layer which consist of the first and the second rows of the transition metals (21-30 and 39-48 atomic numbers), each row generates circles and curves of it's own . Their flows are also based on Archimedes spiral but in opposite directions which their equations can be considered as $r_1 = a\theta$ and $r_2 = a(\pi - \theta)$.

Fig. (14) is about the third and forth rows of the transition metals and Lanthanides and Actinides, the fractal pattern has gotten more complicated. There are two separated curves for Lanthanides and Actinides, and each curve consisted of two Archimedes spirals. The equations two Archimedes spirals for elements of the sixth period (atomic numbers 57-80) can be determined as $r_1 = a|(\theta + \frac{\pi}{2})|$, $\theta \rangle 0$ and $r_2 = a|(\frac{\pi}{2} - \theta)|$, $\theta \rangle \pi$

As for the elements of the seventh period (atomic numbers 89-112), The equations two Archimedes spirals can be written as $r_3 = a|(\theta)|$, $\theta \rangle \frac{\pi}{2}$ and $r_4 = a|(\pi + \theta)|$, $\theta \rangle -\frac{\pi}{2}$

The chemical elements of the periodic table have shown before that there is an relationship between atomic numbers and atomic weights of elements which is based on Sierpinski triangle fractals [12].This work also shows that there are Sierpinski carpet fractals among all the elements. So chemical elements can not be arranged just as a linear system but also they can create fractals and spiral systems.

3.5.2 Comparison with spiral chaos

The flows and the curves in Figs (12), (13) and (14) and the results in those layers can show a spiral chaos. R¨ossler chaos and Shilnikov chaos are the most famous spiral ones. The flows for these two chaos are shown in Fig (15)[13,14].

With addition of new elements with a new period ,the second and middle layer in Fig (13b) remains constant and unchanged, but the flows in Fig (12b) will be continued. Additional elements in Fig (14b) will cause new layers and also a continuing flow (Fig 10). To compare the resulted curves in those layers in Figs (16) with R¨ossler and Shilnikov chaos , the flows must be drawn through connected elements.

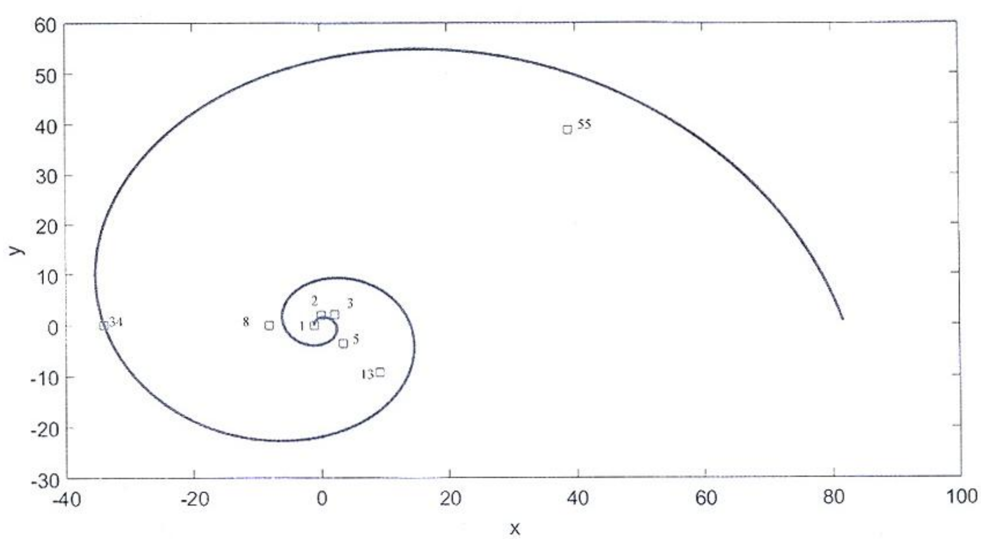


Fig. 11. New version of golden spiral (position)

Fig (17) shows the going flows between the three layers. To draw the flows and connections, Calcium and Strontium in the first layer (left) with atomic numbers 20 and 38 are connected to Scandium and Yttrium which are in the middle layer with atomic numbers 21 and 39, respectively. Also, Barium and Radium with atomic numbers 56 and 88 which are in the first layer are connected to Lanthanum and Actinium which are in the third layer (right) with atomic numbers 57 and 89, respectively.

To draw the reverse or going back flows, Zinc and Cadmium in the middle layer with atomic numbers 30 and 48 are connected to Gallium and Indium which are in the first layer with atomic numbers 31 and 49, respectively. Next, Mercury and Copernicium in the right layer with atomic numbers 80 and 112 are connected to Thallium and Ununtrium which are in the first layer with atomic numbers 81 and 113, respectively. Fig (18) shows going flows (black lines) and reverse flows (blue lines) between these three layers.

All the curves and the flows which are inside and between these three layers together indicate a spiral chaos among the chemical elements. Comparison of the resulted chaos with R¨ossler and Shilnikov chaos in Fig (15), determines this type of chaos is similar to Shilnikov chaos, but it has fundamental differences with Shilnikov chaos.

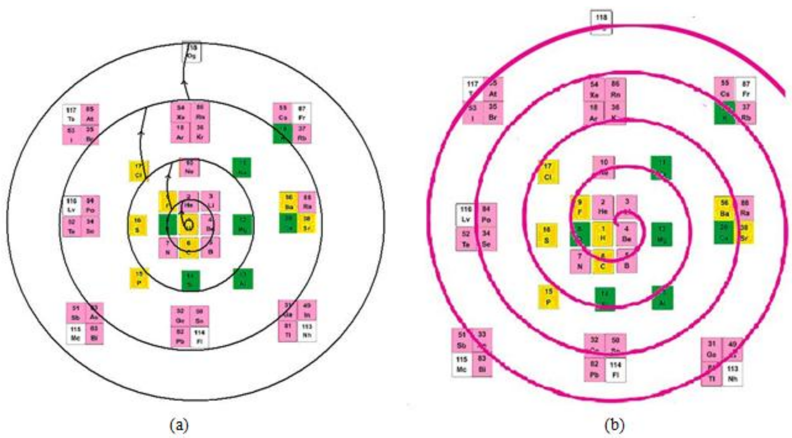


Fig. 12. (a) Circles and (b) the curve of s and p block elements

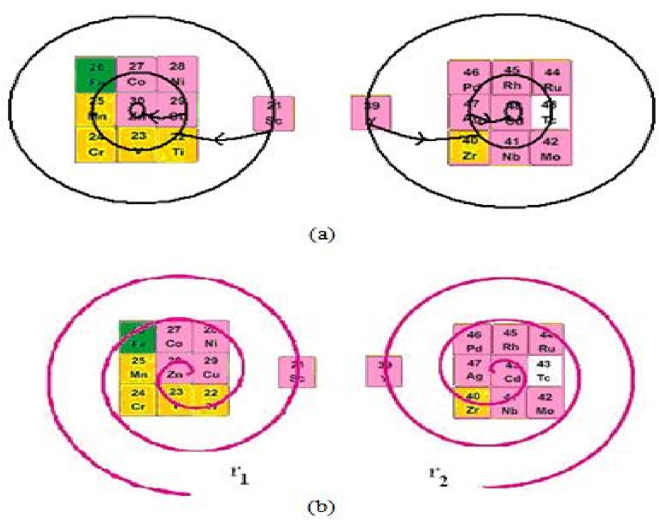


Fig. 13. (a) Circles and (b) curves of transition elements of fourth and fifth periods

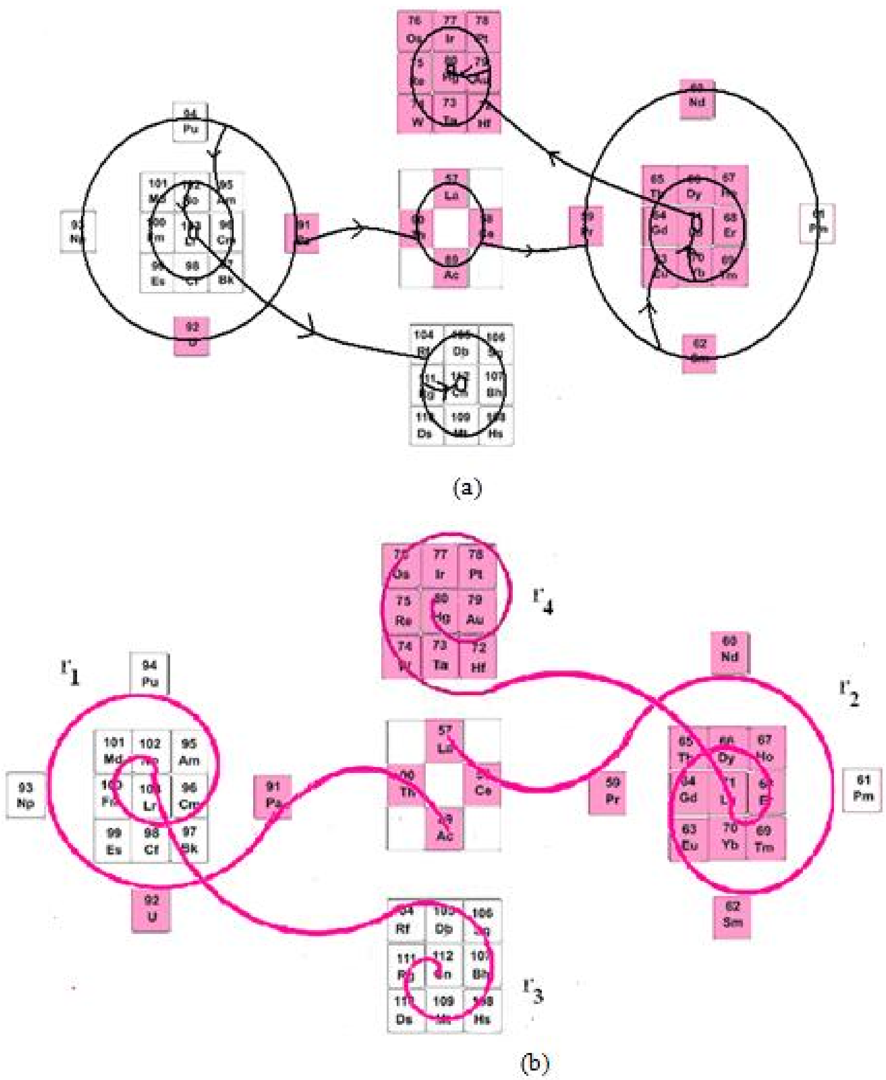


Fig .14. (a) circles and (b) curves of transition elements of sixth and seventh periods

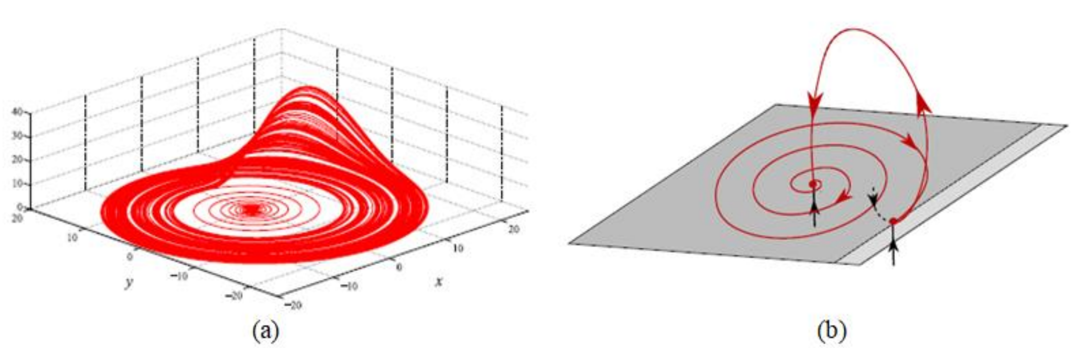


Fig. 15. (a) R¨ossler chaos and (b)Shilnikov chaos[13,14]

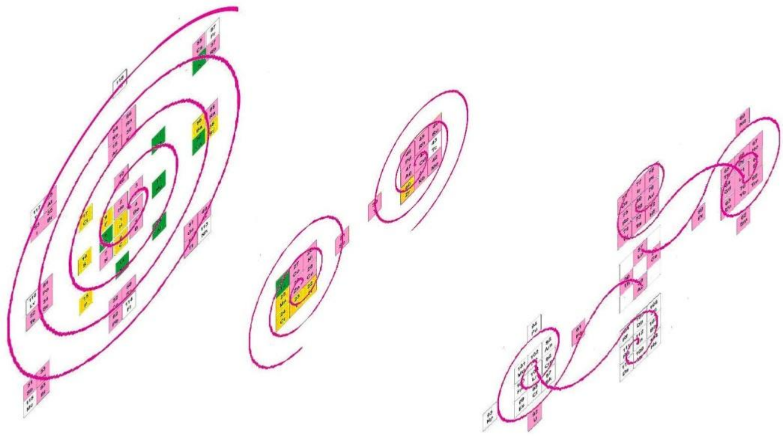


Fig. 16. the three layers and their flows

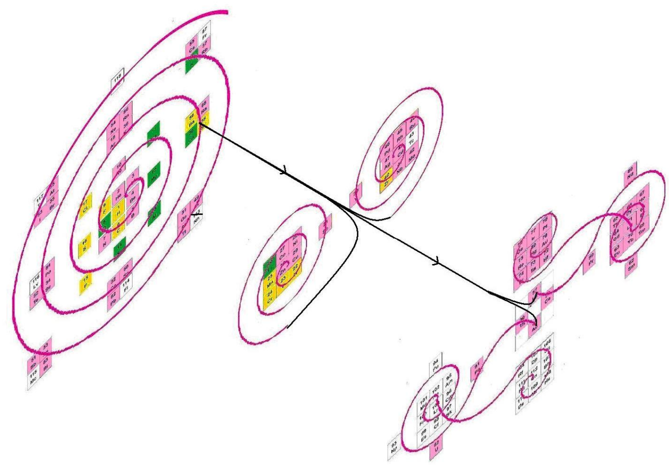


Fig. 17. the going flow among the three layer

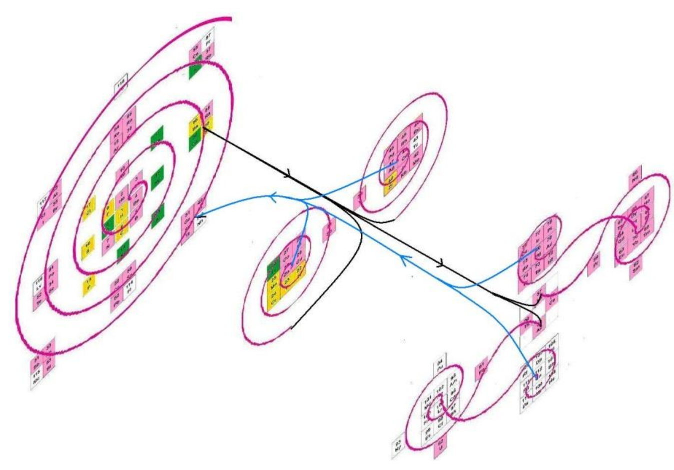


Fig.18. The flows among the three layer and created chaos

First, the equations of the three layers are based on Archimedes spiral equation. Second, the flews in Shilnikov chaos constantly continues but the resulted flews in the three layers are completely different. The curve of the first layer which belongs to s and p block elements, can continues with new elements. But in the third layer, additional new elements will continue the flow and also will make a new layer. The second layer which consisted of the first and second rows of d block elements in Fig (16) remains constant and additional elements will not cause any change in the flow, so the curves and flows in this layer will not be continued and remains constant. Figs (10), (13) and (18) confirm these conclusions.

4. CONCLUSION

The periodic table of the chemical elements is an orderly linear system, but these chemical elements can follow fractal rules and in this work they are arranged as Sierpinski carpet sets. When all 118 elements are set as Sierpinski carpet order, the linear system of chemical elements transforms to three separate layers and spiral systems. Each spiral systems is based on Archimedes spiral equations. By connecting the flows and the curves resulted of these three systems a spiral chaos can be made. In this chaotic system, additional elements of future eighth period will not affect the flows in the middle layer, but the flows and curves of the first and third layers will be continued with more elements and even can create a fourth layer, too. The resulted figures of the chemical elements indicate that the middle layer separates the other two layers and Those two other layers will have their own self-repeatability and self-similarity with additional elements. Chaos theory has applications in a variety of disciplines. This work shows a chaotic system which consists of different parts. One part is a constant layer surrounded by two separate, continual and self – repeatable layers.

More interesting point is that the future- discover elements of the eighth period will follow the Sierpinski carpet pattern to continue the chaos and not stopping it. Fractal sets, equations and chaotic systems of the chemical elements will help chemists to know better about relationship between chemistry and geometry. This author likes to call this chaos which is created in the periodic table of the elements as Hojatkashani chaos.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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