Journal of Advances in Mathematics and Computer Science

37(5): 44-57, 2022; Article no.JAMCS.88388

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

SIR Model for COVID-19 Dynamics Incorporating Clinical Management

Lucian Talu Mayabi [a*], Joyce Kagendo Nthiiri [a], Frankline Tireito [a] and Tecla Sum [a]

[a] *Department of Mathematics, Masinde Muliro University of Science and Technology, P.O Box- 190-50100, Kakamega, Kenya.*

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2022/v37i530452

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/88388

Original Research Article

Received: 10 April 2022
Accepted: 18 June 2022
Published: 29 June 2022

Abstract

In this paper, a mathematical model for COVID-19 disease incorporating clinical management based on a system of Ordinary Differential Equations is developed. The existence of the steady states of the model are determined and the effective reproduction number derived using the next generation matrix approach. Stability analysis of the model is carried out to determine the conditions that favour the spread of COVID-19 disease in a given population. The Disease Free Equilibrium is show to be locally asymptotically stable when $R_e < 1$ and the Endemic Equilibrium is locally asymptotically stable when $R_e > 1$. The Disease Free Equilibrium is shown not to be globally asymptotically stable using a technique by Castillo Chavez and the Endemic Equilibrium is shown to be globally asymptotically stable by means of Lyapunov's direct method and LaSalle's invariance principle. This implies that COVID-19 disease transmission can be kept low or manageable with the incorporation of clinical management. Sensitivity analysis of the model is carried out by use of the normalised forward sensitivity index (elasticity) which shows that the higher the rates of clinical management the lower the rate of infection. Numerical simulations carried out using MATLAB software showed that with high success of clinical management, there is low contact rate and low prevalence rate of the disease in the population.

Keywords: COVID-19; clinical management; effective reproduction number.

**Corresponding author: E-mail: talumayabi@gmail.com;*

1 Introduction

COVID-19 is a respiratory disease whose first outbreak was reported in Wuhan City, Hubei Province, China, on 31^{st} December, 2019. The disease is caused by a virus called Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2)[1]. COVID-19 mainly spreads via respiratory droplets produced when an infected person coughs, sneezes or exhales. The droplets are transferred from both symptomatic and asymptomatic people to others through close personal contact such as touching or shaking hands, touching an object or surface with the virus on it, and subsequently touching one's face (that is eyes,ears,nose or mouth) with contaminated hands. People can also get the infection through breathing in droplets coughed by someone who is infected [2]. According to [3], sometimes the spread of COVID-19 can occur through airborne transmission whereby an infected person releases small droplets and particles with the virus that could stay in the air for minutes to hours and infect people who are further than 6 feet away from them. This is more often possible in enclosed spaces with poor ventilation and when you are exposed for a longer period of time. Airborne transmission of COVID-19 may also be possible in specific circumstances and settings in which procedures or support treatments that generate aerosols are performed such as endotracheal intubation. The incubation period of COVID-19 is on average 4-5 days, but can be as long as 14 days [4].

According to the World Health Organization (WHO) report [5], worldwide, as of March 7^{th} 2022, since the outbreak of COVID-19 was reported in December, 2019, there have been 448 million confirmed positive cases of COVID-19, including 6.01 million deaths and over 93.3 million who have recovered. In Africa, as of August 8^{th} 2021, 7,075,119 confirmed positive cases have been reported with a total of over 3.27 million deaths. In Kenya, as of March 7^{th} 2022, since the first case of COVID-19 was reported in March, 2020, 323,000 positive cases have been confirmed with a total of 5,641 death cases and over 188,000 who have recovered.

Clinical manifestations of COVID-19 are generally milder in children compared with adults. Infection categories range from asymptomatic infection, mild illness, moderate illness, severe illness, to critical illness. Asymptomatic infection stage individuals test positive for SARS-CoV-2 using a virological test (an antigen test). Mild illness individuals have various signs and symptoms such as low grade fever (about 100 degrees Fahrenheit), dry cough, fatigue, headache, gastrointestinal upset (vomiting and diarrhea), muscle pain, sore throat, nasal congestion/runny nose, loss of taste, and loss of smell. Moderate illness individuals show evidence of some difficulties in breathing, chills, deep cough and high grade fever of greater than 100.4 degrees Fahrenheit. Severe illness individuals have chest discomfort problems, eye problems, bluish face/lips and some dermatological manifestations including rash on skin and discoloured lesions of the fingers/toes. Critical illness individuals have respiratory failure, septic shock, and/or multiple organ dysfunction [6, 7, 8, 9].

Diagnosis allows suspected people to understand whether they are infected or not. Additionally, it helps them to receive the care they need and take measures to cut back the probability of infecting others. If a person develops symptoms of COVID-19 disease, it means they have been exposed to the virus, s(he) should consult a doctor. The doctor will decide whether to conduct tests for COVID-19 based on individuals signs and symptoms. Screening is done at the first point of contact at the emergency department. Diagnosis of COVID-19 disease is typically made using Polymerase Chain Reaction (PCR) testing via nasal swab. However, because of false-negative test result rates of SARS-CoV-2 PCR testing of nasal swabs, laboratory findings may also be used to make a presumptive diagnosis. Specific diagnosis is by specific molecular tests on respiratory samples (throat swab/ nasopharyngeal swab/ sputum/ endotracheal aspirates), stool and in several cases, the blood. CT scans is more sensitive and specific [10, 11, 12, 13].

Various containment measures can be used to address, prevent and combat the spread of COVID-19 disease. These include; physical distancing of at least one meter apart between individuals, isolation of infected individuals, quarantine of exposed individuals, cessation of movements temporarily in the most affected areas, lockdown, cleaning and disinfecting frequently touched surfaces and objects, and temporal closure of social amenities [14]. It is believed that people with Underlying health conditions such as pulmonary hypertension, diabetes, cardiovascular disease, chronic respiratory disease, immune compromised status, cancer, and obesity are at a high risk of becoming positive with COVID-19. Additionally, people who are categorized under other factors such as Demographic factor (Older cohort), Organ failure specifically the kidney, Weakened immune system (organ transplant patients, cancer patients and HIV/AIDS patients) and Clinical factors (decreased lymphocytes, and increased leucocytes) too are at a high risk of testing positive on COVID-19 [15].

In [11, 16], Clinical management and treatment for COVID-19 disease include measures taken to cater for individuals both in presymptomatic and symptomatic infection stages. Patients with mild illness may not initially require hospitalization and therefore can manage their illness at home in accordance to home-based care measures such as getting enough bed rest, staying in a separate isolated room, eating a balanced diet and maintaining water electrolyte balance. Patients with severe illness should be given supplemental oxygen immediately, they should then be closely monitored for signs of clinical deterioration such as rapidly progressive respiratory failure and respond immediately with supportive care interventions. In patients with critical illness, Endotracheal intubation should be performed by a trained and experienced provider using airborne precautions, use of a conservative fluid management strategy where there is no evidence of shock, implementation of Mechanical ventilation, and ExtraCorporeal Membrane Oxygenation (ECMO) in addition to the use of Repurposable drugs. Repurposable drugs on the other hand involves the use of Antiviral drugs, Anti-Inflammatory drugs, Antiparasitic drugs and Interferon therapy. According to [17], recent studies done by Centres for Disease Control and Prevention (CDC), and WHO show that we have; Pfizer-BioNTec, Stupnik V, VaxZevria (previously known as AstraZeneca), Sinovac (CoronaVac), and Johnson & Johnson vaccines.

The high numbers of COVID-19 related cases have led to the development of a couple of mathematical models to explain the various containment measures in the spread of COVID-19 disease for instance [18, 19, 20, 21].

Epidemiological research on COVID-19 has been marked with much activity for the past months. A mathematical model [22] of COVID-19 transmission dynamics and control strategy has been done. In this model, the spread of the disease largely depend on the contact rate, therefore, effort should be made to minimize unnecessary contact with COVID-19 infected individuals to reduce an outbreak. Additionally, it's possible to reduce the infection rate by adherence to appropriate containment measures such as physical distancing of at least one meter apart between individuals, isolation of infected individuals, quarantine of exposed individuals, cessation of movements temporarily in the most affected areas, lockdown, cleaning and disinfecting frequently touched surfaces and objects, and temporal closure of social amenities.

In [23], Optimal Control Analysis was done where Public Health Education was one of the parameters of investigation. In this model, public health education effort involved educating the public through social media, television, radio, and traditional rulers in the community on how to observe the various containment measures put in place to combat COVID-19 disease spread. It was deduced that awareness corrects the misconceptions and encourages the public to take appropriate control measures. Additionally, it reduces the number of exposed and infected individuals drastically within a short time.

In [24], A Mathematical Model for SARS-CoV-2 Infection with Treatment was developed. It was observed that the length of the dosing interval and the drug dose play a very decisive role to control and eradicate COVID-19 disease infection. Additionally, if the treatment regimen is not adjusted properly, the therapy is not effective at all.

Clinical Management, which is a combination of both preventive and treatment measures has been presented theoretically. Additionally, a couple of Mathematical Models have been developed to study the impact of various intervention strategies. There was need for combined efforts/measures to combat COVID-19 disease. Thus, this study develops a mathematical model of COVID-19 disease incorporating clinical management based on a system of Ordinary Differential Equations. The objectives for this study were: To develop a Mathematical Model incorporating clinical management, to carry out stability analysis of the developed model in order to study the long term behavior of solutions about the equilibrium points and to perform numerical simulations of the developed model in order to evaluate the impact of clinical management in the spread of COVID-19 disease.

2 The Model

We formulate a model in which the total human population under study denoted as N(t) comprises of four categories namely; Susceptible population (S), Asymptomatic infected population (I_A), Symptomatic infected population (I_S), and Recovered population (R). Susceptible individuals are entirely recruited by immigration at per capita rate Λ. Susceptible individual transits from compartment S to compartment I_A at the rate λ (the effective force of infection). Both I_A and I_S are infectious. An individual could move from compartment I_A to I_S at the rate τ (transition rate from I_A to I_S) if they show symptoms. Upon treatment, the infected individual recovers at the rate β (recovery rate). μ and δ are the natural mortality rate and disease mortality rate respectively. The model assumed; the recruitment into the population is Λ, individuals recover at a constant rate β, the model assumes a heterogenous population approximation, and an individual gains temporal immunity after recovery.

The Model can then be expressed by the following system of Ordinary Differential Equations that represent the rates of changes of population densities of S,I_A,I_S,R with respect to time.

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S - \lambda S, \\
\frac{dI_A}{dt} &= \lambda S - \mu I_A - \tau I_A, \\
\frac{dI_S}{dt} &= \tau I_A - \mu I_S - \delta I_S - \beta I_S, \\
\frac{dR}{dt} &= \beta I_S - \mu R.
\end{aligned} \tag{2.1}$$

with the initial conditions

$$\begin{aligned}
S(0) &= s_0 \geq 0, \\
I_A(0) &= I_{A_0} \geq 0, \\
I_S(0) &= I_{S_0} \geq 0, \\
R(0) &= r_0 \geq 0.
\end{aligned} \tag{2.2}$$

where the total population at time t is given by,

$$N(t) = S(t) + I_A(t) + I_S(t) + R(t) \tag{2.3}$$

The force of infection λ^c is defined by:

$$\lambda^c = \frac{\epsilon\phi(1-\kappa)(\eta I_A + I_S)}{N}. \tag{2.4}$$

such that ϵ is the probability rate of acquiring COVID-19, ϕ is the contact rate with infected individuals, and η is the modification parameter that denotes the most infectious group of individuals in the population. Asymptomatic infected individuals show high 'silent' risk of transmission. Let κ be the probability of success of clinical management against COVID-19 such that $0 \leq \kappa < 1$.

3 Model Analysis

We study the positivity and boundedness of the solutions of the system (2.1) with initial conditions (2.2). Since we are dealing with human population, all population compartments should be nonnegative $\forall$ t > 0 in the feasible region Γ where $S, I_A, I_S, R \in \Gamma \subset \Re^4_+$ and it can be shown that N(t)$< \Lambda$ $+\omega$ for all large t where ω is an arbitrary small positive constant. Thus the set of solutions are ultimately bounded. Thus, the model equation (2.1) is epidemiologically well posed in the region Γ.

The effective reproduction number, R_e, computed using the next generation matrix approach for equation (2.1) is given by;

$$R_e = \frac{(\mu+\delta+\beta)(\epsilon\phi\eta - \epsilon\phi\eta\kappa) + \epsilon\phi\tau(1-\kappa)}{(\mu+\tau)(\mu+\delta+\beta)} \tag{3.1}$$

3.1 Existence of equilibrium points

The disease free equilibrium points of the model are its steady state solutions in the absence of infection or disease that is when I_A=I_S=R=0 while endemic equilibrium points of the model are its steady state solutions in the presence of infection or disease in a given population that is when S, I_A, I_S, R$\neq$ 0.

To obtain the equilibrium points for the model we set the right hand side to zero,

$$\begin{aligned} \Lambda - \mu S - \lambda S &= 0 \\ \lambda S - \mu I_A - \tau I_A &= 0 \\ \tau I_A - \mu I_S - \delta I_S - \beta I_S &= 0 \\ \beta I_S - \mu R &= 0. \end{aligned} \tag{3.2}$$

At disease free equilibrium, we have that I_A=I_S=R=0. Therefore the DFE is given by;

$$DFE = (\frac{\Lambda}{\mu}, 0, 0, 0). \tag{3.3}$$

To calculate endemic equilibrium, we have that S, I_A, I_S, R$\neq 0$

$$\begin{aligned}
S^* &= \frac{\Lambda N}{\epsilon\phi(1-\kappa)([\frac{\eta}{\tau+\mu}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})]+[\frac{\tau}{(\tau+\mu)(\mu+\delta+\beta)}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})])+\mu N}\\
l_A^* &= \frac{1}{\tau+\mu}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})\\
I_S^* &= \frac{\tau}{(\tau+\mu)(\mu+\delta+\beta)}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})\\
R^* &= \frac{\beta\tau}{\mu(\tau+\mu)(\mu+\delta+\beta)}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})
\end{aligned}$$

Therefore the endemic equilibrium (EE)is given by;

$$EE=(S^*,I_A^*,I_S^*,R^*). \tag{3.4}$$

3.2 Local stability analysis of the disease free equilibrium (DFE)

The Jacobi of the system (2.1) is given by;

$$J=\begin{bmatrix}
-(\mu+\frac{\epsilon\phi(\eta I_A+I_S-\kappa\eta I_A-\kappa I_S)}{N}) & 0 & 0 & 0\\
\frac{\epsilon\phi(\eta I_A+I_S-\kappa\eta I_A-\kappa I_S)}{N} & -(\frac{\epsilon\phi\eta S(1-\kappa)}{N}+\mu+\tau) & \frac{\epsilon\phi S(1-\kappa)}{N} & 0\\
0 & \tau & -(\mu+\delta+\beta) & 0\\
0 & 0 & \beta & -\mu
\end{bmatrix}$$

We now evaluate the Jacobi at DFE. We have that;

$$J_{DFE}=\begin{bmatrix}
-\mu & 0 & 0 & 0\\
0 & -(\frac{\epsilon\phi\eta(1-\kappa)\frac{\Lambda}{\mu}}{N}+\mu+\tau) & \frac{\epsilon\phi(1-\kappa)\frac{\Lambda}{\mu}}{N} & 0\\
0 & \tau & -(\mu+\delta+\beta) & 0\\
0 & 0 & \beta & -\mu
\end{bmatrix}$$

We notice that;

$$\lambda_1 = -\mu, \text{is an eigen value}$$

Further, we have;

$$J_{DFE}=\begin{bmatrix}
-(\frac{\epsilon\phi\eta(1-\kappa)\frac{\Lambda}{\mu}}{N}+\mu+\tau) & \frac{\epsilon\phi(1-\kappa)\frac{\Lambda}{\mu}}{N} & 0\\
\tau & -(\mu+\delta+\beta) & 0\\
0 & \beta & -\mu
\end{bmatrix}$$

Clearly,

$$\lambda_2 = -\mu$$

We now analyze a 2 by 2 matrix given by;

$$J_{DFE}=\begin{bmatrix}
-(\frac{\epsilon\phi\eta(1-\kappa)\frac{\Lambda}{\mu}}{N}+\mu+\tau) & \frac{\epsilon\phi(1-\kappa)\frac{\Lambda}{\mu}}{N}\\
\tau & -(\mu+\delta+\beta)
\end{bmatrix}$$

Clearly, the trace is negative. We now find the determinant where we have;

$$(\mu+\delta+\beta)(\frac{N\epsilon\phi\eta\Lambda}{\mu}+\mu+\tau)-\frac{\tau N\epsilon\phi\Lambda(1-\kappa)}{\mu} \quad > \quad 0$$

For the determinant to be positive, we have that;

$$(\mu+\delta+\beta)(\frac{N\epsilon\phi\eta\Lambda}{\mu}+\mu+\tau) \quad > \quad \frac{\tau N\epsilon\phi\Lambda(1-\kappa)}{\mu}$$

or

$$\frac{\tau N\epsilon\phi\Lambda(1-\kappa)}{(\mu+\delta+\beta)(N\epsilon\phi\eta\Lambda+\mu+\tau)} \quad < \quad 1$$

Incorporating R_e, we have that;

$$\frac{1}{\epsilon\phi\eta(\mu+\delta+\beta)}(R_e-\frac{\epsilon\phi\eta(1-\kappa)}{\mu+\tau}) \quad < \quad 1$$

which is negative provided that $R_e<1$. The determinant of the jacobian matrix at DFE remains positive provided that $R_e<1$, the model has a stable DFE when $R_e<1$. This implies that the DFE is locally asymptotically stable. The prevalence ratio of COVID-19 infectives in the population would decrease and approach the value 0, therefore, the COVID-19 disease epidemic would die out of the population with time.

3.3 Local stability analysis of the endemic equilibrium (EE) of the model

A disease is endemic in a population if it persists in the population.

$$J_{EE}=\begin{bmatrix} -(\mu+\frac{\epsilon\phi(1-\kappa)(\eta b+c)}{N}) & 0 & 0 & 0 \\ \frac{\epsilon\phi(1-\kappa)(\eta b+c)}{N} & -(\frac{\epsilon\phi\eta(1-\kappa)a}{N}+\mu+\tau) & \frac{\epsilon\phi(1-\kappa)a}{N} & 0 \\ 0 & \tau & -(\mu+\delta+\beta) & 0 \\ 0 & 0 & \beta & -\mu \end{bmatrix}$$

where;

$$a \quad = \quad \frac{\Lambda N}{\epsilon\phi(1-\kappa)([\frac{\eta}{\tau+\mu}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})]+[\frac{\tau}{(\tau+\mu)(\mu+\delta+\beta)}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})])+\mu N}$$

$$b \quad = \quad \frac{1}{\tau+\mu}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})$$

$$c \quad = \quad \frac{\tau}{(\tau+\mu)(\mu+\delta+\beta)}(\frac{\mu N}{\epsilon\phi(1-\kappa)}+\eta\Lambda N+\frac{N\tau\mu}{\epsilon\phi(1-\kappa)})$$

We notice that;

$$\lambda_1 \quad = \quad -(\mu+\frac{\epsilon\phi(1-\kappa)(\eta b+c)}{N})$$

Further;

$$J_{EE}=\begin{bmatrix} -(\frac{\epsilon\phi\eta(1-\kappa)a}{N}+\mu+\tau) & \frac{\epsilon\phi(1-\kappa)a}{N} & 0 \\ \tau & -(\mu+\delta+\beta) & 0 \\ 0 & \beta & -\mu \end{bmatrix}$$

Clearly;

$$\lambda_2 \quad = \quad -\mu$$

We now analyze the 2 by 2 matrix given by;

$$J_{EE} = \begin{bmatrix} -(\frac{\epsilon\phi\eta(1-\kappa)a}{N} + \mu + \tau) & \frac{\epsilon\phi(1-\kappa)a}{N} \\ \tau & -(\mu + \delta + \beta) \end{bmatrix}$$

The trace is negative. We now find the determinant where we have;

$$(\mu + \delta + \beta)(\frac{\epsilon\phi\eta(1-\kappa)a}{N} + \mu + \tau) - \frac{\tau\epsilon\phi(1-\kappa)a}{N} \quad > \quad 0$$

For determinant to be positive, we have that;

$$(\mu + \delta + \beta)(\frac{\epsilon\phi\eta(1-\kappa)a}{N} + \mu + \tau) \quad > \quad \frac{\tau\epsilon\phi(1-\kappa)a}{N}$$

Incorporating R_e, we have that;

$$[\frac{R_e}{\epsilon\phi\eta(1-\kappa)(\mu + \delta + \beta)} - \frac{1}{(\epsilon\phi\eta(1-\kappa))(\mu + \tau)}] \quad > \quad 1$$

The determinant of the jacobian matrix at EE remains positive provided that $R_e > 1$. The model has a locally asymptotically stable EE when $R_e > 1$. This implies that COVID-19 infection incidences are increasing thus, COVID-19 disease epidemic would certainly develop.

3.4 Global stability of the disease free equilibrium (DFE)

For global stability of the DFE, the technique used by Castillo was employed [25]. There are two conditions that if met, the global asymptotic stability of the disease free equilibrium point is guaranteed. Equation (2.1) can be written in the form:

$$\begin{aligned} \frac{dX}{dt} &= K(X, Z), \\ \frac{dZ}{dt} &= G(X, Z) \end{aligned} \tag{3.5}$$

where X$\in \Re^2$ and X=(S,R) denotes the number of uninfected individuals and Z$\in \Re^2$ where Z{2} denotes the number of infected individuals. DFE = ($\frac{\Lambda}{\mu}$,0,0,0) where

$$X^* \quad = \quad \frac{\Lambda}{\mu}$$

conditions (3.6) may be met to guarantee global asymptotic stability.

$$\begin{aligned} \frac{dX}{dt} &= K(X, 0), \text{ DFE is globally asymptotically stable,} \\ G(X, Z) &= AZ - \hat{G}(X, Z), \hat{G}(X, Z) \geq 0 \forall (X, Z) \in \Gamma \end{aligned} \tag{3.6}$$

where A=$D_z G(DFE, 0)$ is an M Matrix and Γ is the region where the model has biological meaning.

Theorem

lf system (2.1) satisfies conditions (3.6), then the fixed point DFE = ($\frac{\Lambda}{\mu}$,0,0,0) is a globally asymptotically stable equilibrium of the system (2.1) provided that $R_e < 1$.

Proof

Consider K(X,0)=$(\Lambda - \mu S)$ and G(X,Z)= AZ- $\hat{G}(X, Z)$ where

$$A = -(\mu + \delta + \beta) \tag{3.7}$$

and

$$\hat{G}(X, Z) = -\frac{\epsilon\phi(1-\kappa)(\eta I_A + I_S)S}{N} \tag{3.8}$$

Since all the conditions in Equation (3.6) are not satisfied because $\hat{G}(X, Z) < 0$, the DFE may not be globally asymptotically stable.

This implies that we anticipate an outbreak of COVID-19 disease when particular conditions which favour the outbreak of the disease are prevailing.

3.5 Global stability of the endemic equilibrium (EE)

The global stability of the endemic equilibrium may be obtained by means of Lyapunov's direct method and LaSalle's invariance principle [26]. Consider the non-linear Lyapunov function

$$L : (S, I_A, I_S, R) \in \Gamma \in \Re^4_+ : S, I_A, I_S, R > 0 \tag{3.9}$$

where

$$L : (S, I_A, I_S, R) = \lambda(S - S^* - S^* \log \frac{S}{S^*}) + b + c + d \tag{3.10}$$

in which b,c and d are given by the following respectively.

$$\begin{aligned} b &= \lambda(I_A - I_A^* - I_A^* \log \frac{I_A}{I_A^*}) \\ c &= \lambda(I_S - I_S^* - I_S^* \log \frac{I_S}{I_S^*}) \\ d &= \lambda(R - R^* - R^* \log \frac{R}{R^*}) \end{aligned}$$

where L in C^1 in the interior of Γ. EE is the global minimum of L: (I_A,I_S,R)=0. The time derivative of L is given by

$$\frac{dL}{dt} = \dot{L} = \lambda(1 - \frac{S^*}{S})\frac{dS}{dt} + \lambda(1 - \frac{I_A^*}{I_A})\frac{dI_A}{dt} + \lambda(1 - \frac{I_S^*}{I_S})\frac{dI_S}{dt} + \lambda(1 - \frac{R^*}{R})\frac{dR}{dt} \tag{3.11}$$

with the derivatives of equation S, I_A, I_S, R defined in equation (3.11) and by using the EE points, $\dot{L} < 0$. We see that $\dot{L} = 0$ iff S=S^*, I_A=I_A^*, I_S=I_S^* and R=R^*. Thus the largest compact invariant set in (S,I_A,I_S,R) ∈ Γ :L=0 is the singleton EE , where EE is the endemic equilibrium. Thus, EE is globally asymptotically stable in the interior of the region Γ.

This implies that clinical management produces desired results in terms of COVID-19 intervention. Therefore, COVID-19 disease transmission levels can be kept quite low or manageable incorporating clinical management.

3.6 Sensitivity Analysis

Sensitivity analysis of R_e with respect to the model parameter (κ) is carried out in order to determine the impact of clinical management in COVID-19 transmission. To perform sensitivity analysis, we used the normalised forward sensitivity index (elasticity). The normalised forward sensitivity index of the effective reproduction number R_e in equation (3.1) with respect to clinical management (sensitivity parameter) κ is given by;

$$\Gamma_{\kappa}^{R_e} = \frac{\partial R_e}{\partial \kappa} \times \frac{\kappa}{R_e} = 1 \tag{3.12}$$

Meaning that the higher the rates of clinical management, the lower the rate of infection.

3.7 Parameter values for the Model

Table 1. Parameter values for the model

Parameter symbol	Description	Value	Source
λ	Force of infection	0.5	Varies
μ	Natural mortality rate	0.02	Estimated
τ	Transition rate from I_A to I_S	0.002	Estimated
δ	Disease mortality rate	0.03	Estimated
β	Recovery rate	0.02	Estimated
ϵ	Rate of acquiring COVID-19	0.003	[20]
θ	Contact rate with infected people	0.005	Varies
η	Modification Parameter	0.005	Varies
κ	Probability of success of CM	$0< \kappa < 1$	By definition

3.8 Numerical Simulation

The NonStandard Finite Difference Method (NSFD) is used for the numerical solution of the model equation (2.1). NSFD is an iterative method in which we get closer to solution through iteration. Let the nonstandard ODEs be given as:

$$y_k^{'} = f[t, y_1, y_2, ..., y_n] \tag{3.13}$$

where k= 1, 2,..,n. Incorporating NSFD method for numerical solution for equation (2.1), we have that;

$$\begin{aligned}
\frac{S_{n+1} - S_n}{\Delta t} &= \Lambda - \mu S_n - \lambda S_n, \\
\frac{l_{An+1} - l_{An}}{\Delta t} &= \lambda S_n - \mu I_{An} - \tau I_{An}, \\
\frac{l_{Sn+1} - l_{Sn}}{\Delta t} &= \tau I_{An} - \mu I_{Sn} - \delta I_{Sn} - \beta I_{Sn}, \\
\frac{R_{n+1} - R_n}{\Delta t} &= \beta I_{Sn} - \mu R_n.
\end{aligned} \tag{3.14}$$

We use the following initial conditions assuming that we are working on a population (N) estimate of 1000 individuals; S(0)=100, $I_A(0)$=200, $I_S(0)$=500 and R(0)=200. Table 1 represent the corresponding predicted parameter values using Kenya COVID-19 data. The figure below represents the projections of data fitted on model equation (2.1).

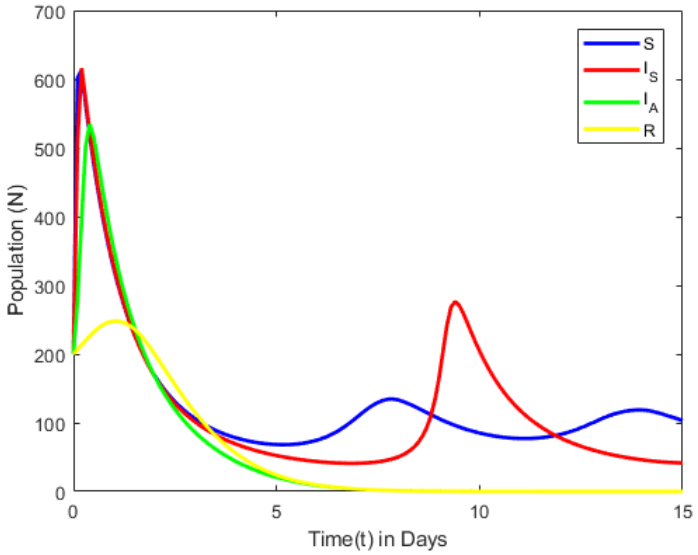

Fig. 1. Population when $R_e = 2.875$

Fig. 1 shows the solutions of system (2.1) when $R_e > 1$, in the presence of infection. DFE is unstable and we observe that the epidemic will certainly develop.

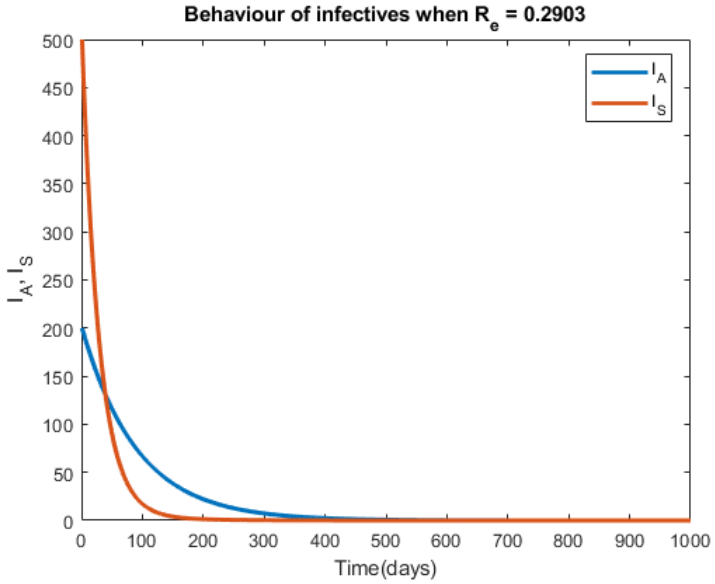

Fig. 2. Behaviour of infectives when $R_e = 0.2903$

Fig. 2 shows the solution of system (2.1) when the effective reproduction number, $R_e < 1$. The number of infectives decrease and approach the value 0. This implies that the disease would die out

of the population with time. With high success of clinical management, there is low contact and prevalence rates hence infective in the population decreases sharply over time. On contrary, low clinical management leads to high contact rate with the infected and hence a high disease prevalence in the population.

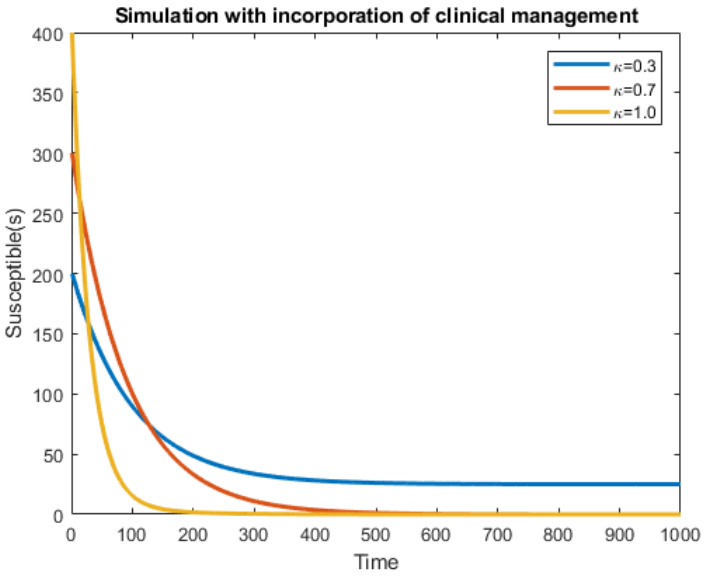

Fig. 3. Simulation with incorporation of clinical management when $\kappa = 0.3, 0.7, 1.0$

Fig. 3 shows graph of susceptible(s) against time in days. With high success of clinical management there is low contact and prevalence rates hence the susceptible in the population decreases with time. On the other hand, when clinical management is low, the number of susceptible individuals will be high.

4 Discussion

The purpose of the study was to explore the need for combined efforts to combat COVID-19 disease. DFE is locally asymptotically stable when $R_e < 1$ and this implies that the prevalence ratio of COVID-19 infectives in the population would decrease and approach the value 0, therefore, the COVID-19 disease epidemic would die out of the population with time. EE is locally asymptotically stable when $R_e > 1$ implying that COVID-19 infection incidences are increasing thus, COVID-19 disease epidemic would certainly develop.

From the numerical simulations, we observe that with high success of clinical management, there is low contact rate and low prevalence rate hence the infective in the population decreases sharply over time. With low levels of clinical management there is high contact rate with the infected and hence a high disease prevalence in the population.

Acknowledgement

I sincerely acknowledge with thanks the Department of Mathematics staff members, Masinde Muliro University of Science and Technology for their support during this study.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] W.H.O. "Corona virus disease outbreak review," World Health Organization; 2020.

[2] Liao X, et al. Community Transmission of Severe Acute Respiratory Syndrome Coronavirus 2. Shenzhen, China; 2020.

[3] Modes of Transmission of Virus causing COVID-19 : Implications for IPC precaution recommendations.
Available: who.int/news-room/commentaries/detail.

[4] Lauer SA. The Incubation Period of Coronavirus Disease 2019 (COVID-19) From Publicly Reported Confirmed Cases: Estimation and Application. Ann Intern Med, 2020;172:577-582.

[5] JHU CSSE. COVID-19 DATA, 2021.

[6] Allotey J, et al. Clinical Manifestations of coronavirus disease 2019:living systematic review and meta-analysis. BMJ. 2020;370.

[7] CDC (Centers for DiseasExtraCorporeal Membrane Oxygenation (ECMO) in addition to the use of Repurposable drugse Control and Prevention), Symptoms of Coronavirus
Available: https://www.cdc.gov/coronavirus/2019-ncov/about/symptoms.html.

[8] "Clinical Spectrum of SARS-CoV-2 Infection," April 2021, see covid19treatmentguidelines.nih.gov/overview/clinical-spectrum/.

[9] Wei Tan, et al. Skin Manifestations of COVID-19: A Worldwide Review. Jaad International, Elsevier; 2020.

[10] Abdul H. A review of COVID-19 diagnosis. Treatments and Prevention; 2020.
DOI:10.14744/ejmo.90853.

[11] Clinical Management of SARI when COVID-19 is suspected. Interim guidance; 2020. Available: See https://doi.org/10.1007/s12098-020.03263-6.

[12] Tanu S. A Review of COVID-19. The Indian Journal of Pediatrics 2020;87(4):281-286.

[13] Joost, W.W. Pathophysiology, Trasmission, Diagnosis and Treatment of COVID-19. JAMA Review, 2020;816.

[14] Xiaoyan, Z., and Yuxuan, W. Prevention and Control mechanism for coronavirus disease 2019 epidemic at the primary level: Perspective from China. Cambridge University Press; 2020.

[15] Anderson RM, Fraser C, Ghani A, Donnelly A, Riley S, Ferguson M. Epidemiology, transmission dynamics and control of SARS: the 2002–2003 epidemic. Philos. Trans. R. Soc. Lond., B, Biol. Sci. 2004;359:1091–1105.

[16] Hasanul BS, Nahida HN, Mohammad SH. Stopping the COVID-19 Pandemic: A Review on the Advances of Diagnosis, Treatment and Control Measures. Hindawi;2020:Article ID 9121429.

[17] Frequently asked questions about Covid-19 vaccine; 2020
see Available: https://www.westerncape.gov.za/department-of-health/frequently-asked-questions/frequently-asked-questionsabout-covid-19 vaccine.

[18] Anwar Zeb et al. Mathematical Model for Coronavirus disease 2019 containing isolation class. Hindawi BioMed Research International. 2020;Article ID 3452406.

[19] Biao Tang et al. The effectiveness of quarantine and isolation. International Journal of Infectious Diseases. 2020;95:288-293.

[20] Nyambadza F, Chirove F, Chukwu CW, Visaya. Modelling the potential impact of social distancing on COVID-19 epidemic in SA. Hindawi Journal. 2020;Article ID 5379278.

[21] Richard JHS, et al. A Modelling framework to assess the likely effectiveness of facemask in combination with 'lock-down' in managing the COVID-19 Pandemic. Royal society publishing organisation; 2020.

[22] Jumanne MM, Naboth SZ. Mathematical Model of COVID-19 Transmission Dynamics and Control Strategies. Journal of Applied and Computational Mathematics, 2020; 9:1.

[23] Chinwendu E. Madubueze, Sambo Dachollom and Isaac Obiajulu Onwubuya, Controlling the Spread of COVID-19: Optimal Control Analysis. 2020;2020:Article ID 6862516.

[24] Amar NC, Fahad AB. A Model for SARS-CoV-2 Infection with Treatment. 2020;Article ID 1352982.

[25] Castillo-Chavez, C., Zhilan, F., Wenzhan, H. On the computation of RO and its role in global stability in Mathematical approaches for emerging and re-emerging infectious diseases. An introduction. Institute for Mathematics and its Applications. 2002;125:229-250.

[26] Leon De CV. Constructions of Lyapunov functions for classics SIS, SIR and SIRS epidemic model with variable population size. Unite Academica de Mathematicas, Universidadn Autonoma de Guerrere, Mexico Facultad de Estudios Superiores Zarogoza, UNAM, Mexico; 2009.

[27] Ngonghala CN, Iboi E, Eikenberry S, et al., Mathematical assessment of the impact of non-pharmaceutical interventions on curtailing the 2019 novel coronavirus. Mathematical Biosciences. 2020;325:Article 108364.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/88388