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Entropy Corrected $f(G)$Gravity Using Sharma Mitall Holographic Dark Energy Model

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

By using a power law modified gravity model *f*(*G*) together with Sharma Mitall Holographic Dark Energy model (*SMHDE*) a more understanding of universe behavior is obtained. In our model we reconstruct the Hubble parameter, consequently we verified the cosmic accelerating expansion using the deceleration, equation of state parameters and the diagnostic state finder parameter. The stability of the model is achieved using the square speed of sound parameter. For completeness we study the energy conditions and the results obtained support the idea of universe accelerating expansion behavior. By considering a quantum correction for both power law and logarithmic corrected entropy, the validity of the generalized second law of thermodynamics is studied.

Keywords: Sharma Mitall entropy; dark energy; f(G)gravity model.

1. INTRODUCTION

Studying the accelerated expansion of the universe is one of the main tasks in cosmology in the last two decades. In 1998 the first observation for that expansion is recognized and that opens the window for deeper understanding for the nature of the universe. From that time

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cosmologists and mathematicians proposed many models to understand what is going on in our universe and trying to give some answer to some open questions [1–4]. Dark energy (DE) with a negative pressure is may be considered as the main source of accelerated expansion behavior of the universe. To explore the nature of this energy some models are established like scalar field models and dynamical dark energy models [5–11].

By using torsion (T), curvature scalars (R) and GaussBonnet invariant (G) a new approach has been proposed to explore the mystery of this energy by modifying the standard gravity theories to form what called modified gravity theories [12, 13]

An important model that is widely used to understand DE is the holographic dark energy model (HDE) [14]. This model is created based on the holographic principle [15], in which the degrees of freedom of the system are scaled using the boundary area rather that its volume [16]. The holographic energy density is close to the dark energy density if we take the infrared (IR) cutoff to be equal to the size of universe. The correspondence between holographic principle and various dark energy models are studied in many pervious work [17–22].

In last years a new formula for entropy are used together with holographic principle to establish new holographic dark energy models, namely: Tsallis HDE, Renyi HDE and Sharma Mitall Holographic Dark Energy (SMHDE) to understand cosmic the accelerating expansion of our universe assuming it is filled by both interacting dark energy and cold dark matter [23–28]. What is really interested in SMHDE that it shows a good agreement with universe expansion and shows a type of cosmos stability, the dynamics of SMHDE and the growth of energy density are controlled by its free parameters. In this study we will examine the effect of changing these free parameters together Gauss-Bonnet modified gravity model *f*(*G*) on the growth of cosmic parameters and the thermal behavior of the considered model [29]. The structure of the paper is as follows, in (II) section we will consider the model, in section (III) we will study the diagnostic state finder parameter, in (IV) stability of the model is considered using the square speed of sound then energy conditions and entropy of the model are studied and finally the conclusion is present.

2. THE MODEL

The 4-dimension action of *f*(*G*) theory is given by [30,31]:

$$S = \frac{1}{k^2}\int dx^4\sqrt{-g}\left(\frac{r}{2} + f(G)\right) + S_m, \tag{1}$$

where, r is the Ricci scalar, $k^2 = 1$ is a coupling constant, $S_m$is the matter action and G is the Gauss Bonnet term [30]:

$$G = r^2 - 4r_{\mu\nu}r^{\mu\nu} + r_{\mu\nu\lambda\sigma}r^{\mu\nu\lambda\sigma}, \tag{2}$$

where, $r^{\mu\nu}$ and $r_{\mu\nu\lambda\sigma}$ are the Ricci and Riemann tensors, respectively. The corresponding field equation is given by [30]:

$$r_{\mu\nu} - \frac{1}{2}rg_{\mu\nu} = T_{\mu\nu}^{eff}, \tag{3}$$

where, T^{eff} is the effective energy momentum tensor.From action variation and by using the Friedman fieldequation $3H(t)^2 = \rho + \rho_m$, $2H^{\cdot}(t) + 3H(t)^2 = -(P + P_m)$, where dot represents the first derivative with respect tocosmic time , H is the Hubble parameter.

We can write the energy density and pressure for *f*(*G*) model as [32]:

$$\rho = -24H(t)^3 f_{GG}(t)\dot{G}(t) + G(t)f_G(t) - f(t), \tag{4}$$

and,

$$P = 8H(t)^2 f_{GGG}(t)\dot{G}(t)^2 + f_{GG}(t)\big(8H(t)^2\dot{G}(t)16H(t)\dot{G}(t)\dot{H}(t) + 16Ht3Gt - GtfGt + f(t), \tag{5}$$

where, f_G , f_{GG} and f_{GGG} are the first, second and third derivatives of $f(G)$ with respect to $G(t)$respectively and $G(t)$ is given by [32]:

$$G(t) = 24H(t)^2\big(\dot{H}(t) + H(t)^2\big), \tag{6}$$

In the current study we are going to consider the power law form of *f*(*G*) theory [33], namely:

$$f(G) = \eta G(t)^{m+1}, \tag{7}$$

where, η and m are some arbitrary parameters, that way one can write:

$$f_G = \eta(m+1)G(t)^m, \tag{8}$$

$$f_{GG} = \eta m(m+1)G(t)^{m-1}, \tag{9}$$

$$f_{GG} = \eta(m-1)m(m+1)G(t)^{m-2}, \tag{10}$$

The importance of the power law model comes from two points, that it fits the observation and gives the ability to establish the connection between both inflation and late time acceleration and also we notice that the singularity in big rip theory dose not appears in this model [34].

In the mid seventies of last century Sharma and Mittal proposed two parametric generalized entropy which is given by [25]:

$$S_{SM} = \frac{1}{J}\left(\left(1+\frac{\delta A}{4}\right)^{\gamma} - 1\right), \tag{11}$$

where, J is a free parameter, $\gamma = \frac{J}{\delta}$and δ is a real number, and A is the horizon area. By using the holographic principle, we can relate the ultraviolet cutoff and the infrared cutoff with the entropy of the system by using [35]:

$$\Lambda^4 \leq \frac{S}{L^4}, \tag{12}$$

Following the HDE model, the dark energy is corresponding to ultraviolet cutoff ($\rho \sim \Lambda^4$). If we take the infrared cutoff L to be corresponding to Hubble parameter (H) i.e $L = \frac{1}{H} = \sqrt{A/_{4\pi}}$ [36]. Eqs.(11)and (12) can be written as [25]:

$$\rho = \frac{3c^2 H(t)^4\left(\left(\frac{\delta\pi}{H(t)^2}+1\right)^{\gamma}-1\right)}{8\pi J}, \tag{13}$$

also, we can define the fractional energy density as:

$$\Omega = \frac{\rho}{3H(t)^2}, \tag{14}$$

where, c is a numerical parameter. Eq.(13) represents the Sharma Mitall Holographic Dark Energy (SMHDE), we can recover the ordinary HDE model by choosing $J = \delta$. Now by combing Eqs.(4), (6) and (13) and after some calculations, we can reconstruct the Hubble parameter for our model. Now by considering the relation between the cosmic time t and red-shift z, namely $t = \frac{2}{((z+1)^2+1)H_o}$and by choosing the parameters values as $m = -\frac{1}{2}$, $H_o = 67$, $\eta = 3$, $\alpha = 3$ and $\beta = -3$ for the analysis in this work. In Fig. 1 (a) we study the growth of Hubble parameter H against the red shift z we notice a deceasing behavior over the considered z range. In Fig. 1 (b) we study of expansion of the universe by considering the deceleration parameter $q = -\frac{\dot{H(t)}}{H(t)^2} - 1$ we notice that q decreases with the increase in z stays in negative that indicates accelerating expansion behavior in (c) the positive evolution of Ω'verify the observation of the growth of dark energy with time.

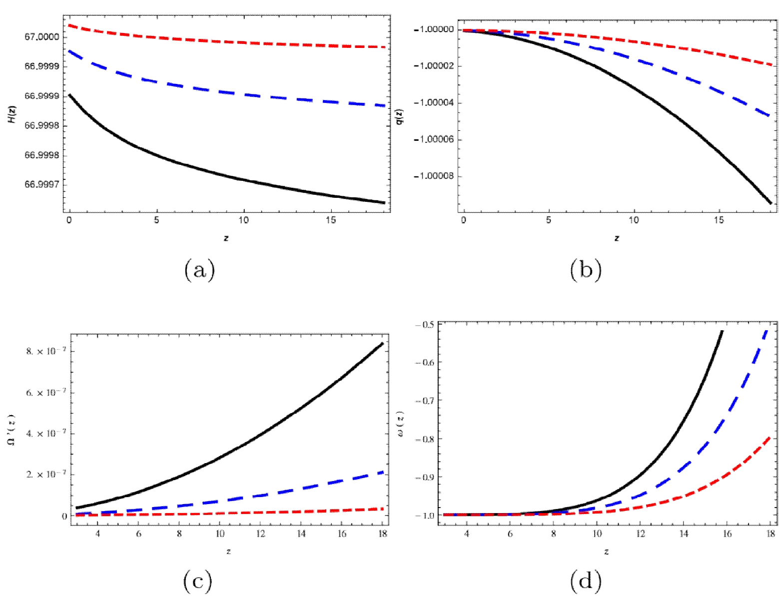

Fig. 1. (a) The growth of *H* (b) the *q*(c) the Ω (d) the*EoS*as function of red-shift *z* . Red, blue and black lines correspond to $\gamma = -400, -800$ *and* − 2000 for the *f*(*G*) model

For Sharma Mitall Holographic Dark Energy dominate universe, one can write the equation of continuity assuming non interacting case between universe components as:

$$\rho^{\cdot} + 3H(\rho + p) = 0, \tag{15}$$

that way we can write the equation of state parameter as:

$$\omega = \frac{P}{\rho} = -\frac{\rho^{\cdot} - 3H\rho}{3H\rho}, \tag{16}$$

To know which phase of the expanding universe is considered by our model equation of state parameter is studied. In Fig. 1 (d) the evolution of equation of state parameter is studied showing a quintessence mode since $\omega > -1$.

3. DIAGNOSTIC OMEGA Om

In this section, we are going to study the diagnostic-omega *Om*, it is a geometrical state finder parameter with no timederivative of Hubble parameter that makes it easier in treatment rather than the other diagnostic parameters which contain time derivative orders for Hubble parameter. The diagnostic-*Om* is given by the formula [37]:

$$Om = \frac{\frac{H(t)^2 - 1}{H_0{}^2}}{(z+1)^3 - 1}. \tag{17}$$

This Om is established to understand the difference between ΛCDM and the other DE models. In Fig. 2 (b) we study the growth of Omas a function of *z*, actually it is the slope of *Om*-*z* graph that is used to understand the expansion behavior of the universe. The positive slope indicates the phantom model while the negative one leads to quintessence behavior.

4. SQUARE SPEED OF SOUND v^2

To examine the stability behavior of our model, we use a parameter called the square speed of sound which is denoted as v^2 and is defined by the following formula [38]:

$$v^2 = \frac{\dot{P}}{\dot{\rho}} = \dot{\omega}\frac{\rho}{\dot{\rho}} + \omega. \tag{18}$$

By using this formula, we can check the stability of the model, simply for negative values of v^2 >0 the model is unstable while for positive values the model is stable. In Fig. 2 (c) we study the growth of v^2 as a function of *z* although this study is independent on the variation of *γ* parameter, following the graph trajectories we observe a positive behavior for v^2 and this analysis leads to the stability of the considered model.

5. ENERGY CONDITIONS

For completeness, we will study the energy conditions in this section, these conditions are vital in understanding the features of some important cosmological theorems about black holes [39]. The energy conditions are weak energy condition (*WEC*) $\rho \geq 0, \rho + p \geq 0$, strong energy condition (*SEC*) $\rho + 3p \geq 0$, $\rho + p \geq 0$. and dominant energy condition (*DEC*) $\rho - |p| \geq 0$. The verification of some black holes models is related to the energy conditions [40]. In Fig. 3. The growth of energy conditions (b) *WEC* (c) *SEC* (d) *DEC* against the red shift is studied, we

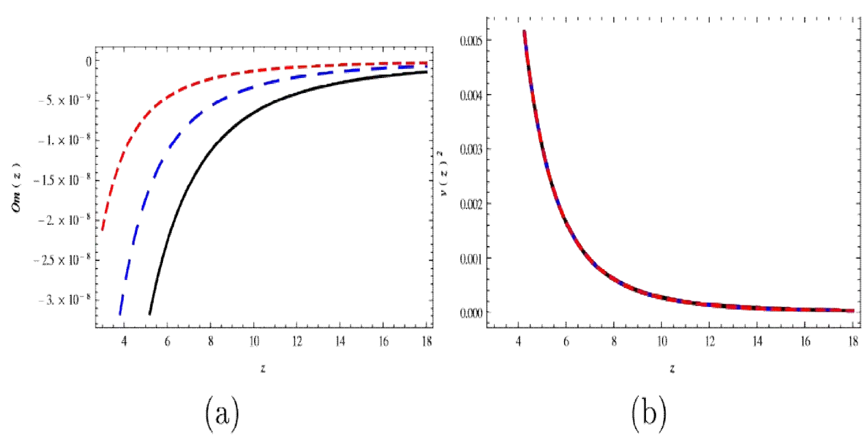

Fig. 2. (a) The growth of the *Om* (b) the v^2 against the red-shift *z* Red, blue and black lines correspond to *γ* = -400, $-8\ 00$*and* -2000 for the $f(G)$ model

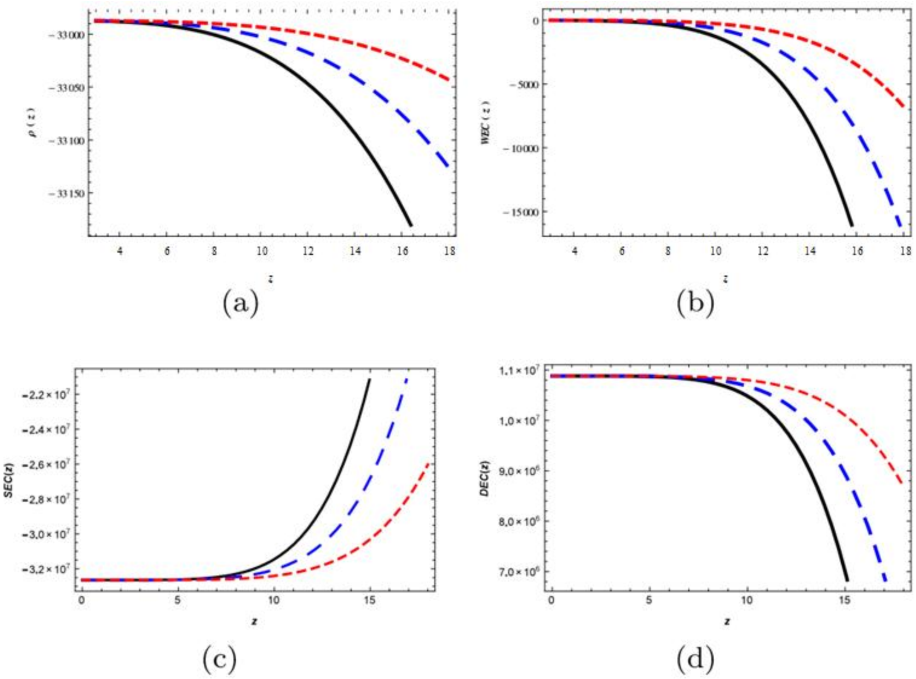

Fig. 3. The growth of (a) *WEC* (b)*SEC* (c) *DEC* as function of red-shift *z* Red, blue and black lines correspond to γ = -400, $-8\ 00$*and* -2000 for the $f(G)$ model

notice that *WEC* is violated over the *z* range for $m = -1/2$ and that is related phantom regime in which the EoS is less than -1. We observe in Fig. 3. (c) the violation of *SEC* that is leading to the existence of dark energy, moreover the *DEC* is also satisfied in Fig. 3. (d) that way the considered model is fitting the observation by assuming the accelerating expansion nature of the universe.

6. CORRECTED ENTROPY PARAMETERS

In the generalized second law of thermodynamics, the sum of the entropy of the horizon and the entropy of total matter inside the horizon does not decrease with time. The first law of thermodynamics states $-dE = TdS$, where $S = A/4G$is the entropy area relation and *T* is the Hawking temperature. Actually some corrections is considered to the entropy using the entanglement of quantum fields in and out the horizon, like a power-corrected area term in the entropy expression [41,42].

The entropy relation is modified that leads to the modification of power law corrected entropy (*PLCE*) and logarithmic corrected entropy (*LCE*) [43,44], namely:

$$S_P = \frac{A}{4G}\left(1 - K_\alpha A^{1-\frac{\alpha}{2}}\right), \tag{19}$$

where,

$$K_\alpha = \frac{(4\pi)^{\frac{\alpha}{2}-1}\alpha H_0^{2-\alpha}}{4G}, \tag{20}$$

where, $A = \pi {R_X}^2$ is the horizon area, R_X is the radius of an arbitrary horizon, α is a dimensionless constant which should be positive and$r_c$is the cross scale. The curvature correction in the Einstein-Hilbert action is created due to quantum corrections into the entropy-area relationship. This leads to the logarithmic corrected entropy (*LCE*) [45]:

$$S_L = \frac{A}{4G} + \beta \log(\frac{A}{4G}) + \chi, \tag{21}$$

where, β and χ are some arbitrary constants whose exact values are not yet known. These corrections arise due to mass, charge, quantum and thermal equilibrium fluctuations in loop quantum gravity. We assume that our system in thermal equilibrium is bounded by Hubble horizon which is given by:

$$R_X = \frac{1}{H} \tag{22}$$

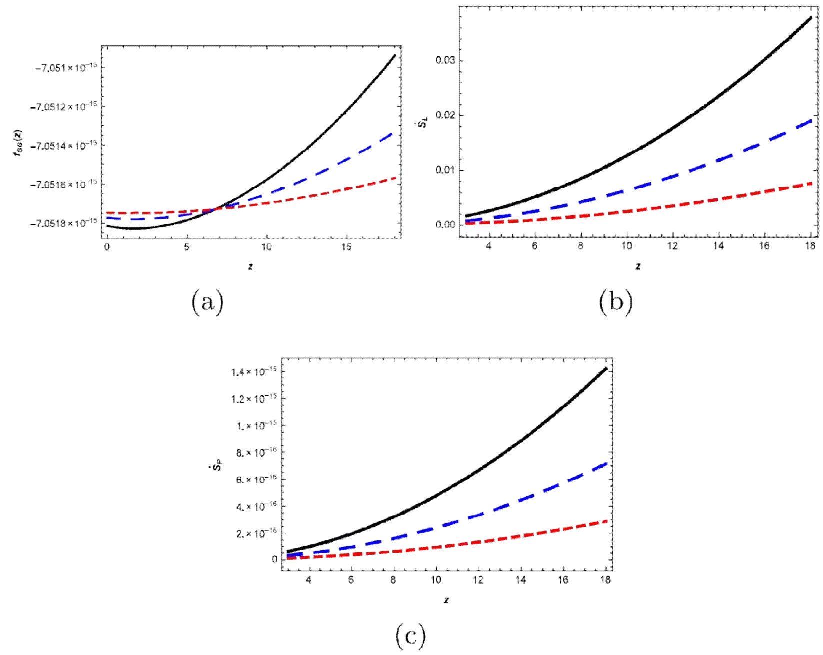

Fig. 4.(a) The growth of f_{GG} (b)power law entropy *SP* and (c)logarithmic entropy *SL*. Red, blue and black lines correspond to $\gamma = -400, -800$ *and* -2000. for the *f*(*G*) model

By using Eq.(10) by taking the first time derivative of Eq.(19) and Eq.(21) one can write after some algebraic steps the corrected entropy for both logarithmic and power law:

$$\dot{S_P} = \frac{2\pi R_X}{G}(\dot{R_X} f_G - 6H\dot{H} f_{GG} R_X)(1-(4\pi)^{1-\frac{\alpha}{2}}(2-\frac{\alpha}{2})K(f_G {R_X}^2)^{1-\frac{\alpha}{2}}), \quad (23)$$

and,

$$\dot{S_L} = 2\left(\frac{\beta}{f_G R_X} + \frac{\pi R_X}{G}\right)\left(\dot{R_X} f_G - 6H\dot{H} f_{GG} R_X\right). \quad (24)$$

In Fig. 4 (a) the evolution of the f_{GG} against the red shift showing f_{GG} is negative over the considered z range. In Fig. 4 (b) and (c) we study the evolution of *GSLT* over the assumed red shift range we notice that *PLCE* has an increasing behavior over the z-range if α is positive. Also the *LCE* stays in the positive level if the arbitrary parameter β <0, that way, the *GSLT* is satisfied over the whole z range if α >0 and β <0 for both power law and logarithmic entropy for the Hubble cut-off.

7. CONCLUSION

We study the evolution of Gauss-Bonnet modified gravity model $f(G)$ in the framework work of Sharma Mitall Holographic Dark Energy model (*SMHDE*), the main cosmological parameters are calculated. The reconse crated Hubble parameter *H* shows a decreasing behavior over the considered *z* range, for certain γ values *H* tends to reach a constant value close to the present value. We investigate the deceleration parameter *q* which shows a negative behavior that supports the recent observations which assume the accelerated expansion nature of the universe. The equation of state parameter ω is considered for our model, it starts from -1 and increases with the increase of z value consequently quintessence mode is observed. The growth of the dark energy is observed using Ω' >0 parameter indicating that dark energy increases with time. The analysis diagnostic state finder parameter *Om* is considered, the slope of *Om* − *z* plane indicates the universe mode, if slope >0 leads to phantom regime but if the slope <0 we get quintessence like behavior. what is interested here that our model is stable because the square speed of sound parameter v^2 is positive over the whole range of z values with an independent behavior on γ values and this is one of the a benefit of using *f*(*G*) theory in our model. As a further remark, we have investigated the energy conditions namely *SEC*, *WEC* and *DEC* we observe the violation of both *SEC*, *WEC* and the validity of *DEC* and that leads to the accelerated expansion behavior. For thermal analysis we studied the generalized second law of thermodynamics *GSLT* for our reconstructed *f*(*G*) gravity model in *SMHDE* universe. By considering the corrected entropy version, we studied the power law corrected entropy and logarithmic entropy assumed by the entropy-area equation. We notice both *LCE* and *PLCE* are hold. Indeed, we addressed that the validity of *GSLT* for that model over the considered red shift range. Hence, exploring the nature of Sharma Mitall Holographic Dark Energy model in the framework of Gauss-Bonnet modified gravity model shows some novel investigations in *SMHDE* like the stability is achieved, the well known cosmological parameters all leads in an excellent way to what is supported by recent observations. In fact, the study of theories of modified gravity using HDE models opens the way to a deeper understanding of our universe.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Riess AG, et al. Observational Evidence from Supernovae for anccelerating Universe and a Cosmological Constant J. Astron. 1998;116:1009.
2. WMAP Collaboration Collaboration, E. Komatsu. et al. Astrophys. J. Suppl. 2011;192:18.
3. Ichiki K et al. Primordial neutrinos, cosmological perturbations ininteracting dark-energy model: CMB and LSS. J. Cosmol. Astropart. Phys. 2008;06:005.
4. Astier P, et al. The Supernova Legacy Survey: Measurement of Ω_M , Ω_Λ and w from the first year data set" Astron. Astrophys. 2006;447:31.
5. Armendariz-Picon C. Essentials of k-essence et al. Phys. Rev. D. 2001;63: 103510.
6. Ratra B, Peebles PJ. Cosmological consequences of a rolling homogeneous scalar field" Phys. Rev. D. 1988;37: 3406.
7. Sen A. Rolling Tachyon. J. of High Energy Physics. 2002;4:48.
8. Caldwell RR. A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state. Phys. Lett. B. 2002;545: 23.
9. Arkani-Hamed N, et al. “Ghost inflation” J. Cosmol. Astropart. Phys. 2004;4:1.
10. Anisimov A. “B-inflatio” J Cosmol. Astropart. Phys. 2005;6:6.
11. Kamenshchik A, et al. An alternative to quintessence. Physics Letters B. 2001; 511:265.
12. Wei H, Cai RG. A new model of agegraphic dark energy. Physics Letters B. 2008;660:113.
13. Nojiri S, Odintsov SD. Unified cosmic history in modified gravity: From F (R) theory to Lorentz non-invariant models. Physics report. 2011;505:144.
14. Li M. A model of holographic dark energy. Phys. Lett. B. 2004;603:1.
15. Shuang Wang, Yi ang, Miao Li. Holographic dark energy. Physics Reports. 2017;696:1.
16. Ehsan Sadri, Martiros Khurshudyan. An interacting new holographic dark energy model: Observational constraints” International Journal of Modern Physics D. 2019;28:1950152.
17. Setare MR. Holographic tachyon model of dark energy ”Phys. Lett. B. 2007;653:116 .
18. Setare MR, Saridakis EN. Correspondence between holographic and Gauss–Bonnet dark energy models. Phys. Lett. B. 2008; 670:1.X.
19. Wu, Z-Hong Zhu. Reconstructing f(R) theory according to holographic dark energy. Phys. Lett. B. 2008;660:293.
20. Gong Y. Extended holographic dark energy Physical Review D. 2004;70:6.
21. Zhang J et al. Statefinder diagnosis in a non-flat universe and theholographic model of dark energy. JCAP, 2007.
22. Li M et al. Dark Energy Communications in Theoretical Physics. 2011;51:1 .
23. Tsallis C, Cirto L. Black hole thermodynamicalentropy. Eur. Phys. J. C. 2013;73:2487 .
24. Tsallis C. Possible generalization of Boltzmann-Gibbs statistics” J Statist. Phys. 1988;52:479.
25. Paul J, Thomas PY. Sharma-Mittal Entropy Properties on Record Values. Statics. 2016;76:3.
26. Sayahian Jahromi A. et al. Generalized entropy formalism and a new holographic dark energy model, ph. Letter B. 2018; 780,21.
27. Abdul Jawad. Tsallis, Rényi and Sharma-Mittal Holographic Dark Energy Models in Loop Quantum Cosmology. Symmetry. 2018;10(11):635.

28. Sharma UK, Dubey VC. Exploring the Sharma–Mittal HDE models with different diagnostic tools. Eur. Phys. J. Plus. 2020;135:391.
29. Sayani Maity, Ujjal Debnath. Tsallis, Rényi, Sharma-Mittal holographic and new agegraphic dark energy models in D-dimensional fractal universe” EPJP. 2019; 134:514.
30. Felicea A, Tsujikawab S. Construction of cosmologically viable f(G) gravity models. Physics Letters B. 2009;675:1.
31. Ratbay Myrzakulov, Diego Sáez-Gómez, Anca Tureanu. On the ΛCDM Universe in f(G) gravity. Gen Relativ Gravit. 2011; 43:1671.
32. Nojiri S, Odintsov SD, Oikonomou VK. Modified gravity theories on a nutshell: Inflation, bounce and late-time evolution. Physics Report. 2017;692:104 .
33. Cognola G, Elizalde ES, Nojiri SD, Odintsov, Zerbini S. Class of viable modified gravities describing inflation and the onset of accelerated expansion. Phys. Rev. D. 2008;73:084007.
34. Nojiri S, Odintsov SD, Tretyakov PV, Progr. From inflation to dark energy in the non-minimal modified gravity. Theor. Phys. Suppl. 2008;172:81 .
35. Granda LN, Rojas GD. Modified gravity in the framework of holographic dark energy. Modern physics letter A. 2020;35:2050025.
36. Jawadn A, Azhar N, Rani S. Entropy corrected holographic dark energy models in modified gravity. International Journal of Modern Physics. 2017;26:1750040.
37. Sahni V et al. Two new diagnostics of dark energy. Phys. Rev. D. 2008;78: 103502.
38. sharif M, et al. “New Holographic Dark Energy Model in Brans-Dicke Theory. Symmetry. 2018;10:153 .
39. Aly A, et al. Power Law and Logarithmic Entropy Corrected Ricci Dark Energy Models in Brans-Dicke Chameleon Cosmology. IJTP. 2014;10:153.
40. Santos J, et al. Energy conditions and cosmic acceleration Phys. Rev. D. 2007; 75:083523.
41. Bekenstein JD. Black holes and entropy Phys. Rev. 1973;2333:7.
42. Setare M. Generalized second law of thermodynamics in quintom dominated universe Physics Letters B. 2006;641: 130.
43. Li EK. Generalized holographic Ricci dark energy and generalized second law of thermodynamics in Bianchi Type I universe Gen Relativ. Gravity. 2015;47:136 .
44. Bamba K, Jamil M, Momeni D, Myrzakulov R. Generalized second law of thermodynamics in f(T) gravity with entropy corrections Astrophys. Space Sci. 2013;344:259.
45. Sharif M, Fatima HI. “Thermodynamics with corrected entropies in f(G) gravity Astrophysics and Space Science. 2014; 354, 507.

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