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Generalization of Quantum Mechanical Wave Equation in Spherical Coordinate Using Great Metric Tensors and a Variable Gravitational Scalar Potential

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Authors' contributions

This work was carried out in collaboration among all authors. Author AUM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors AUM, WLL and IIE managed the analyses of the study. Authors AUM, MM and YKK managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this research work, the Riemannian Laplacian operator for a spherical system which varies with time, radial distance and time was obtained using the great metric tensors and a varying gravitational scalar potential. Furthermore the obtained Laplacian operator was used to obtain the generalized quantum mechanical wave equation for particles within this field. The Laplacian operator obtained in this work reduces to the well known Laplacian operator in the limit of c^0, and it contained post Euclid or pure Riemannian correction terms of all orders of c^{-2} . Also the generalized quantum mechanical wave equation obtained, in the limit of c^0 reduces to the well known Schrodinger mechanical wave equation, and in the limit of c^{-2} contained additional correction terms not found in the well known Schrodinger wave equation. Hence the results in this work satisfy the Principle of Equivalence in Physics.

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1. INTRODUCTION

Einstein's contribution to relativity was initially an intuitive approach based on a basic elimination of simultaneousity and a mathematical reformulation using the Lorentz transformation. In this respect Einstein just added some more physics to what Poincaré and Lorentz has done much earlier. However, it was Minkowski who introduced the geometrical ideas and the use of a four-dimensional space with time as the fourth dimension. Einstein took over Minkowski's idea and initiated what we may call the program of geometrizing physics, starting with gravity. Later on Einstein and Hilbert attempted the unification of electro-magnetism and gravity while Kaluza and Klein tried the same using an extra fifth dimension. This may have been the beginning of the higher dimensional space-time theories culminating in super strings, super gravity and the Cantorian space-time theory [1-2].

By replacing Euclidean geometry by curved Riemannian one, Einstein was the first to give gravity a geometrical interpretation as a curvature of space-time due to matter. Einstein neither fixed the topology of his theory nor did he use or was aware of the existence of nonclassical geometry which was in any case in its infancy [1-4]. The possibly only encounter of Einstein with M. S. El Naschie's Cantorian like transfinite geometry was when K. Menger presented a paper in a conference held in his honour [1-16].

Quantum mechanics is undergoing a revolution. Not that its substance is changing, but major developments are placing it in the focus of renewed attention, both within the physics community and among the scientifically interested public. First, wonderfully clever table-top experiments involving the manipulation of single photons, atomic particles and molecules are revealing in an ever-more convincing manner theoretically predicted facts about the counterintuitive and sometimes "spooky" behavior of quantum systems and have led to a renewed interest in the formulation of a strictly physics-based (non-philosophical) foundation of quantum mechanics [5].

The interest in studying the Schrödinger equation with position-dependent *mass* (PDM) has been growing in the last fifty years due to its use in describing some physical phenomena as, e.g., the behaviour of the charge carriers in semiconductor heterostructures, nuclear many body problems, and quantum dots physics, among some others. The PDM concept is, by itself, a fundamental problem which is far from being completely understood. Some contributions have been developed from a theoretical point of view and other approaches to find solutions or to generate exactly solvable problems have been also carried out [6]. A Riemannian laplacian operator in spherical polar coordinate using our recent variable gravitational scalar potential has not been constructed before, therefore in this research work we construct the Riemannian laplacian operator and then test for its sound using Schrodinger wave equation.

2. THEORETICAL FRAMEWORK

The great covariant metric tensors for this distribution of mass or pressure in spherical polar coordinates $f(t,r,\theta)$ constructed by [17-19] are given as

$$g_{00} = \left[1 + \frac{2f(t,r,\theta)}{c^2}\right]. \quad (2.1)$$

$$g_{11} = -\left[1 + \frac{2f(t,r,\theta)}{c^2}\right]^{-1}. \quad (2.2)$$

$$g_{22} = -r^2. \quad (2.3)$$

$$g_{33} = -r^2 \sin^2 \theta. \quad (2.4)$$

$g_{\mu\nu} = 0$, otherwise (2.5)

The great contravarient metric tensors for this field were obtained using Quotient Theorem [17-19] are given as

$$g^{00} = \left[1 + \frac{2f(t,r,\theta)}{c^2}\right]^{-1}. \quad (2.6)$$

$$g^{11} = -\left[1 + \frac{2f(t,r,\theta)}{c^2}\right]. \quad (2.7)$$

$$g^{22} = -\frac{1}{r^2}. \quad (2.8)$$

$$g^{33} = -\frac{1}{r^2 \sin^2 \theta}. \qquad (2.9)$$

$$g^{\mu\nu} = 0 \text{, otherwise} \qquad (2.10)$$

The Riemannian Laplacian operator [18,20] is given as

$$\nabla^2{}_R = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha}\left(\sqrt{g} g^{\alpha\beta} \frac{\partial}{\partial x^\beta}\right) \qquad (2.11)$$

Explicitly, using the great metric tensors for this field, (2.11) is given as

$$\nabla^2{}_R = \frac{2}{r}\left(1+\frac{2}{c^2} f(r,\theta,t)\right)\frac{\partial}{\partial r} + \frac{2}{c^2}\frac{\partial f(r,\theta,t)}{\partial r}\frac{\partial}{\partial r} + \left(1+\frac{2}{c^2} f(r,\theta,t)\right)\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}$$

$$+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{2}{c^4}\left(1+\frac{2}{c^2} f(r,\theta,t)\right)^{-2}\frac{\partial f(r,\theta,t)}{\partial t}\frac{\partial}{\partial t} - \frac{1}{c^2}\left(1+\frac{2}{c^2} f(r,\theta,t)\right)\frac{\partial^2}{\partial t^2}. \qquad (2.12)$$

In our recent article [21], the gravitational scalar potential for this field is given as

$$f\left(t,r,\theta\right) = -\frac{k}{r}\exp\left(t - \frac{r\theta}{c}\right) \qquad (2.13)$$

Where

$$k = GM$$

Simplifying (2.13) to the order of c^{-2} gives

$$f = -\frac{k}{r}\left[1+t+\frac{t^2}{2}\right] + \frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]. \qquad (2.14)$$

Differentiating (2.14) with respect to r and simplifying to the order of c^{-2} gives

$$\frac{\partial f}{\partial r} = \frac{k}{r^2}\left[1+t+\frac{t^2}{2}\right] - \frac{k\theta^2}{2c^2}\left[1+2t-\frac{t^2}{r}\right]. \qquad (2.15)$$

Substituting (2.14) and (2.15) into (2.12) gives

$$\nabla^2{}_R = \frac{2}{r}\left[1-\frac{2k}{c^2 r}\left(1+t-\frac{r\theta}{c}+\frac{t^2}{2}-\frac{tr\theta}{c}+\frac{r^2\theta^2}{2c^2}\right)\right]\frac{\partial}{\partial r} + \left[\frac{2k}{c^2 r^2}\left(1+t+\frac{t^2}{2}\right) - \frac{k\theta}{c^3}\left(\frac{\theta}{c}-\frac{t^2}{r}+\frac{2\theta t}{c}\right)\right]\frac{\partial}{\partial r}$$

$$+\left[1-\frac{2k}{c^2 r}\left(1+t-\frac{r\theta}{c}+\frac{t^2}{2}-\frac{tr\theta}{c}+\frac{r^2\theta^2}{2c^2}\right)\right]\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

$$+\frac{2}{c^4}\left[1-\frac{2k}{c^2 r}\left(1+t-\frac{r\theta}{c}+\frac{t^2}{2}-\frac{tr\theta}{c}+\frac{r^2\theta^2}{2c^2}\right)\right]^{-2}\left[-\frac{k}{r}\left(1+t+\frac{t^2}{2}\right)+\frac{k\theta}{c}\left(1+t+\frac{r\theta}{2}\right)\right]\frac{\partial}{\partial t}$$

$$-\frac{1}{c^2}\left[1-\frac{2k}{c^2 r}\left(1+t-\frac{r\theta}{c}+\frac{t^2}{2}-\frac{tr\theta}{c}+\frac{r^2\theta^2}{2c^2}\right)\right]\frac{\partial^2}{\partial t^2} \qquad (2.16)$$

Simplifying (2.16) to the order of c^{-2} gives

$$\nabla^2{}_R = \left[\frac{2}{r} - \frac{2k}{c^2r^2} - \frac{2kt}{c^2r^2} - \frac{kt^2}{c^2r^2}\right]\frac{\partial}{\partial r} + \left[1 - \frac{2k}{c^2r} - \frac{2kt}{c^2r} - \frac{kt^2}{c^2r}\right]\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}$$
$$+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}. \qquad (2.17)$$

Equation (2.17) is the Riemannian Laplacian operator for this field.

The generalize quantum mechanical wave equation based upon the great metric tensor [22,20] is given as

$$i\hbar\frac{\partial}{\partial t}\psi_R = \hat{H}_R\,\psi_R = E\psi_R. \qquad (2.18)$$

Where,

the Hamiltonian $\hat{H}_R$ is given [20] as

$$\hat{H}_R = m_0c\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}} + m_0\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}}f(r,\theta,t)$$
$$+\left[V_H^{ng}\left(r,t,\underset{-H}{P}\right)\right]. \qquad (2.19)$$

Where,

$\nabla^2{}_R$ the Riemannian Laplacian operator
$\hbar$ is the reduced Plank's constant
$V_H^{ng}\left(r,t,\underset{-H}{P}\right)$ is the general non-gravitational potential energy (based upon Riemannian Geometry),
m_0 non-zero rest mass and c is the speed of light in vacuum.

Applying binomial expansion and ignoring higher terms

$$\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}} = \left(1-\frac{\hbar\nabla^2{}_R}{2m_0{}^2c^2}+...\right) \qquad (2.20)$$

$$\left(1-\frac{\hbar\nabla^2{}_R}{2m_0{}^2c^2}\right) = 1-\left[\frac{\hbar}{m_0{}^2c^2r} - \frac{\hbar k}{m_0{}^2c^4r^2} - \frac{\hbar kt}{m_0{}^2c^4r^2} - \frac{\hbar kt^2}{2m_0{}^2c^4r^2}\right]\frac{\partial}{\partial r}$$
$$-\left[\frac{\hbar}{2m_0{}^2c^2} - \frac{\hbar k}{m_0{}^2c^4r} - \frac{\hbar kt}{m_0{}^2c^4r} - \frac{\hbar kt^2}{2m_0{}^2c^4r}\right]\frac{\partial^2}{\partial r^2} - \frac{\hbar}{2m_0{}^2c^2r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}$$
$$-\frac{\hbar}{2m_0{}^2c^2r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\hbar}{2m_0{}^2c^4}\frac{\partial^2}{\partial t^2}. \qquad (2.21)$$

The first term on the left hand side of (2.19) is given as

$$m_0c\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{2m_0{}^2c^2}\right)\psi_R = m_0c^2\left[1-\frac{k}{c^2r}\left(1+t+\frac{t^2}{2}\right)\right]\left\{1-\left[\frac{\hbar}{m_0{}^2c^2r}-\frac{\hbar k}{m_0{}^2c^4r^2}-\frac{\hbar kt}{m_0{}^2c^4r^2}-\frac{\hbar kt^2}{2m_0{}^2c^4r^2}\right]\frac{\partial}{\partial r}\right.$$

$$-\left[\frac{\hbar}{2m_0{}^2c^2}-\frac{\hbar k}{m_0{}^2c^4r}-\frac{\hbar kt}{m_0{}^2c^4r}-\frac{\hbar kt^2}{2m_0{}^2c^4r}\right]\frac{\partial^2}{\partial r^2}-\frac{\hbar}{2m_0{}^2c^2r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}$$

$$\left.-\frac{\hbar}{2m_0{}^2c^2r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\hbar}{2m_0{}^2c^4}\frac{\partial^2}{\partial t^2}\right\}\psi_R. \tag{2.22}$$

Where,

ψ_R is a wave function of the quantum system To the order of c^{-2} (2.22) reduces to

$$m_0c\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{2m_0{}^2c^2}\right)\psi_R = m_0c^2\psi_R-\frac{m_0k}{r}\left(1+t+\frac{t^2}{2}\right)\psi_R+\left[\frac{\hbar k}{m_0c^2r}\left(1+t+\frac{t^2}{2}\right)+\frac{\hbar k}{m_0c^2r^2}+\frac{\hbar kt}{m_0c^2r^2}\right.$$

$$\left.+\frac{\hbar kt^2}{2m_0c^2r^2}-\frac{\hbar}{m_0r}\right]\frac{\partial\psi_R}{\partial r}+\left[\frac{\hbar k}{2m_0c^2r}\left(1+t+\frac{t^2}{2}\right)+\frac{\hbar k}{m_0c^2r}+\frac{\hbar kt}{m_0c^2r}+\frac{\hbar kt^2}{2m_0c^2r}-\frac{\hbar}{m_0}\right]\frac{\partial^2\psi_R}{\partial r^2}$$

$$+\left[\frac{\hbar k}{2m_0c^2r^3}-\frac{\hbar}{2m_0r^2}\right]\frac{\cos\theta}{\sin\theta}\frac{\partial\psi_R}{\partial\theta}+\left[\frac{\hbar k}{2m_0c^2r^3}-\frac{\hbar}{2m_0r^2}\right]\frac{\partial^2\psi_R}{\partial\theta^2}+\frac{\hbar}{2m_0c^2}\frac{\partial^2\psi_R}{\partial t^2}. \tag{2.23}$$

The second term on the left hand side of (2.19) is given as

$$m_0\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}}f\psi_R = m_0\left[1-\frac{k}{c^2r}\left(1+t+\frac{t^2}{2}\right)\right]\left[1-\left\{\frac{\hbar}{m_0{}^2c^2}-\frac{\hbar k}{m_0{}^2c^4r^2}-\frac{\hbar kt}{m_0{}^2c^4r^2}\right.\right.$$

$$\left.-\frac{\hbar kt^2}{m_0{}^2c^4r^2}\right\}\frac{\partial}{\partial r}-\left\{\frac{\hbar}{2m_0{}^2c^2}-\frac{\hbar k}{m_0{}^2c^4r}-\frac{\hbar kt}{m_0{}^2c^4r}-\frac{\hbar kt^2}{m_0{}^2c^4r}\right\}\frac{\partial^2}{\partial r^2}-\frac{\hbar}{2m_0{}^2c^2r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}$$

$$\left.-\frac{\hbar}{2m_0{}^2c^2r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\hbar}{2m_0{}^2c^4}\frac{\partial^2}{\partial t^2}\right]f\psi_R. \tag{2.24}$$

Simplifying (2.24) to the order of c^{-2} gives

$$m_0\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}}f\psi_R = m_0f\psi_R-\frac{m_0k}{c^2r}\left(1+t+\frac{t^2}{2}\right)f\psi_R-\frac{\hbar}{m_0c^2}\left(f\frac{\partial\psi_R}{\partial r}+\psi_R\frac{\partial f}{\partial r}\right)$$

$$-\frac{\hbar}{2m_0c^2}\left(f\frac{\partial^2\psi_R}{\partial r^2}+\psi_R\frac{\partial^2 f}{\partial r^2}\right)-\frac{\hbar}{2m_0c^2}\frac{\cos\theta}{\sin\theta}\left(f\frac{\partial\psi_R}{\partial\theta}+\psi_R\frac{\partial f}{\partial\theta}\right)-\frac{\hbar}{2m_0c^2r^2}\left(f\frac{\partial^2\psi_R}{\partial\theta^2}+\psi_R\frac{\partial^2 f}{\partial\theta^2}\right). \tag{2.25}$$

To the order of c^{-2},

$$\frac{\partial^2 f}{\partial r^2}=-\frac{2k}{r^3}\left[1+t+\frac{t^2}{2}\right]-\frac{k\theta t}{cr^2}\left[t-\theta+\frac{\theta t}{2}\right]. \tag{2.26}$$

$$\frac{\partial f}{\partial t}=-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]. \tag{2.27}$$

$$\frac{\partial^2 f}{\partial t^2}=-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]. \tag{2.28}$$

$$\frac{\partial f}{\partial \theta}=\frac{k}{c}\left[1+t+\frac{t^2}{2}\right]-\frac{kr\theta}{c^2}\left[1-t\right]. \tag{2.29}$$

$$\frac{\partial^2 f}{\partial \theta^2}=-\frac{kr}{c^2}\left[1+t+\frac{t^2}{2}\right]. \tag{2.30}$$

Substituting (2.26)-(2.30) into (2.25) gives

$$m_0\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}}f\psi_R=-\frac{m_0k}{r}\left(1+t+\frac{t^2}{2}\right)\psi_R-\frac{m_0k\theta}{c}\left(1+t-\frac{r\theta}{2c}\right)\psi_R$$
$$-\frac{m_0k}{c^2r}\left(1+t+\frac{t^2}{2}\right)\left[-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\right]\psi_R-\frac{\hbar}{m_0c^2}\left[\left(-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\right)\frac{\partial\psi_R}{\partial r}+\right.$$
$$\left.\psi_R\left\{\frac{k}{r^2}\left[1+t+\frac{t^2}{2}\right]-\frac{k\theta^2}{2c^2}\left[1+2t-\frac{t^2}{r}\right]\right\}\right]-\frac{\hbar}{2m_0c^2}\left[\left(-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\right)\frac{\partial^2\psi_R}{\partial r^2}+\right.$$
$$\left.\psi_R\left\{\frac{2k}{r^3}\left[1+t+\frac{t^2}{2}\right]-\frac{k\theta t}{cr^2}\left[t-\theta+\frac{\theta t}{2}\right]\right\}\right]-\frac{\hbar}{2m_0c^2}\left[\left(-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\right)\frac{\partial\psi_R}{\partial\theta}+\right.$$
$$\left.\psi_R\left\{\frac{k}{c}\left[1+t+\frac{t^2}{2}\right]-\frac{kr\theta}{c^2}\left[1-t\right]\right\}\right]-\frac{\hbar}{2m_0c^2r^2}\left[\left(-\frac{k}{r}\left[1+t+\frac{t^2}{2}\right]+\frac{k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\right)\frac{\partial^2\psi_R}{\partial\theta^2}+\right.$$
$$\left.\psi_R\left\{-\frac{kr}{c^2}\left[1+t+\frac{t^2}{2}\right]\right\}\right]. \tag{2.31}$$

Simplifying (2.31) to the order of c^{-2} gives

$$m_0\left(1+\frac{2}{c^2}f(r,\theta,t)\right)^{\frac{1}{2}}\left(1-\frac{\hbar\nabla^2{}_R}{m_0{}^2c^2}\right)^{\frac{1}{2}}f\psi_R=\frac{\hbar k}{2m_0c^2r}\left[1+t+\frac{t^2}{2}\right]\frac{\partial^2\psi_R}{\partial r^2}+\frac{\hbar}{m_0c^2r}\left[1+t+\frac{t^2}{2}\right]\frac{\partial\psi_R}{\partial r}+$$
$$\frac{\hbar k}{2m_0c^2r}\left[1+t+\frac{t^2}{2}\right]\frac{\partial\psi_R}{\partial\theta}+\frac{\hbar k}{2m_0c^2r^3}\left[1+t+\frac{t^2}{2}\right]\frac{\partial^2\psi_R}{\partial\theta^2}+\left(1+t+\frac{t^2}{2}\right)\left[\frac{\hbar k}{m_0c^2r^3}+\frac{m_0k^2}{c^2r^2}-\frac{\hbar k}{m_0c^2r^2}-\right.$$

$$\left.\frac{m_0 k}{r}\right]\psi_R - \frac{m_0 k\theta}{c}\left[1+t-\frac{r\theta}{2c}\right]\psi_R. \tag{2.32}$$

Substituting (2.23) and (2.32) into (2.18) and simplifying gives

$$E\psi_R =$$
$$\left[\frac{2\hbar k}{m_0c^2r}\left[1+t+\frac{t^2}{2}\right]-\frac{\hbar}{2m_0}\right]\frac{\partial^2\psi_R}{\partial r^2}+\left[\frac{2\hbar k}{m_0c^2r}\left(1+t+\frac{t^2}{2}\right)+\frac{\hbar k}{m_0c^2r^2}\left(1+t+\frac{t^2}{2}\right)-\frac{\hbar}{m_0 r}\right]\frac{\partial\psi_R}{\partial r}+\left[\left(\frac{\hbar k}{2m_0c^2r^3}-\frac{\hbar}{2m_0r^2}\right)\frac{\cos\theta}{\sin\theta}+\right.$$
$$\left.\frac{\hbar k}{2m_0c^2r}\left(1+t+\frac{t^2}{2}\right)\right]\frac{\partial\psi_R}{\partial\theta}+\left[\left(\frac{\hbar k}{2m_0c^2r^3}\left\{1+t+\frac{t^2}{2}\right\}+\frac{\hbar k}{2m_0c^2r^3}-\frac{\hbar}{2m_0r^2}\right)\right]\frac{\partial^2\psi_R}{\partial\theta^2}+\frac{\hbar}{2m_0c^2}\frac{\partial^2\psi_R}{\partial t^2}+m_0c^2\psi_R-\frac{m_0k}{r}\left(1+t+\frac{t^2}{2}\right)\psi_R+$$
$$\left(1+t+\frac{t^2}{2}\right)\left[\frac{\hbar k}{m_0c^2r^3}-\frac{\hbar k}{m_0c^2r^2}+\frac{m_0k^2}{c^2r^2}-\frac{m_0k}{r}\right]\psi_R-\frac{m_0k\theta}{c}\left(1+t-\frac{r\theta}{2c}\right)\psi_R+\left[V_H^{ng}\left(r,t,\underset{-H}{P}\right)\right]\psi_R. \tag{2.33}$$

Let

$$\alpha=\frac{\hbar k}{m_0c^2},\ \beta=\frac{m_0k}{c^2},\ \delta=\frac{\hbar}{m_0},\ \lambda=1+t+\frac{t^2}{2},\xi=1+t-\frac{r\theta}{2c}$$

Substituting into (2.33) gives

$$E\psi_R=\left[\frac{2\alpha\lambda}{r}-\frac{\delta}{2}\right]\frac{\partial^2\psi_R}{\partial r^2}+\left[\frac{2\alpha\lambda}{r}+\frac{\alpha\lambda}{r^2}-\frac{\delta}{r}\right]\frac{\partial\psi_R}{\partial r}+\left[\left(\frac{\alpha}{2r^3}-\frac{\delta}{2r^2}\right)\frac{\cos\theta}{\sin\theta}+\frac{\alpha\lambda}{2r}\right]\frac{\partial\psi_R}{\partial\theta}$$
$$+\left[\left(\frac{\alpha\lambda}{2r^3}+\frac{\alpha}{2r^3}-\frac{\delta}{2r^2}\right)\right]\frac{\partial^2\psi_R}{\partial\theta^2}+\frac{\delta}{2c^2}\frac{\partial^2\psi_R}{\partial t^2}+m_0c^2\psi_R+$$
$$\lambda\left[\frac{\alpha}{r^3}-\frac{\alpha}{r^2}+\frac{\beta k}{r^2}\right]\psi_R-\beta c\xi\theta\psi_R+\left[V_H^{ng}\left(r,t,\underset{-H}{P}\right)\right]\psi_R. \tag{2.34}$$

For mathematical simplicity, let 2=1, $\mu=\alpha\lambda$, $\rho=\beta\lambda k$, $\Lambda=m_0c^2$, $\tau=\beta c\xi\theta$, $\varpi=\frac{\delta}{c^2}$, hence (2.34) becomes

$$E\psi_R=\left[\frac{\mu}{r}-\delta\right]\frac{\partial^2\psi_R}{\partial r^2}+\left[\frac{\mu}{r}+\frac{\mu}{r^2}-\frac{\delta}{r}\right]\frac{\partial\psi_R}{\partial r}+\left[\left(\frac{\alpha}{r^3}-\frac{\delta}{r^2}\right)\frac{\cos\theta}{\sin\theta}+\frac{\mu}{r}\right]\frac{\partial\psi_R}{\partial\theta}$$
$$+\left[\left(\frac{\mu}{r^3}+\frac{\alpha}{r^3}-\frac{\delta}{r^2}\right)\right]\frac{\partial^2\psi_R}{\partial\theta^2}+\varpi\frac{\partial^2\psi_R}{\partial t^2}+\left[\Lambda+\frac{\mu}{r^3}+\frac{\rho-\mu}{r^2}-\tau\right]\psi_R+\left[V_H^{ng}\left(r,t,\underset{-H}{P}\right)\right]\psi_R. \tag{2.35}$$

Where,

$\rho-\mu$ is the orbital angular momentum, the other dimensionless parameters serve as important elements in finding energy eigenvalue and eigenfunctions.

Equation (2.35) is our generalize time independent Schrodinger wave equation based upon the great metric tensor.

3. CONCLUSION

Interestingly, the Laplacian operator obtained in this work (2.17) reduces to the well known Laplacian operator in the limit of c^0, it also contains post Euclid or pure Riemannian correction terms of all orders of c^{-2}. Instructively, our obtained result satisfies the Principle of Equivalence in Physics.

The Riemannian Laplacian operator obtained in this work could be used in the generalization of Maxwell Theory of Electromagnetism in this field based upon Riemannian geometry. It could also be used in the study of gravitoelectric and gravitormagnetic coupling.

Further the result (2.35) is the generalized time independed quantum mechanical wave equation based upon the great metric tensor, in the limit of c^0 it reduces to the well known Schrodinger mechanical wave equation, and the limit of c^{-2} it contains additional correction terms not found in the well known Schrodinger wave equation. Hence this result also satisfies the Principle of Equivalence in Physics.

Our obtained results differ from [23] in the sense that, [23] uses golden metric tensors, secondly, the gravitational scalar potential used in their work was not specified, lastly the Hamiltonian used in this work is the first general Hamiltonian operator given by [18] which differs from the one used in [23].

The solution to (2.35) will give the Riemannian wave function, the energy and other thermodynamic properties, which is an open door for further research.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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