

## Research Article

# Soliton Molecules and Full Symmetry Groups to the KdV-Sawada-Kotera-Ramani Equation

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By  $N$ -soliton solutions and a velocity resonance mechanism, soliton molecules are constructed for the KdV-Sawada-Kotera-Ramani (KSKR) equation, which is used to simulate the resonances of solitons in one-dimensional space. An asymmetric soliton can be formed by adjusting the distance between two solitons of soliton molecule to small enough. The interactions among multiple soliton molecules for the equation are elastic. Then, full symmetry group is derived for the KSKR equation by the symmetry group direct method. From the full symmetry group, a general group invariant solution can be obtained from a known solution.

## 1. Introduction

Soliton molecules, also known as multisoliton complexes, are the bound states of solitons which exhibit molecule-like behavior [1]. Investigation on soliton molecules provides a direct route to study the interactions between solitary waves, and the formation and dissociation of soliton molecules are closely related to subjects such as soliton collision, soliton splashing, soliton rains, and the trapping of solitons. Besides, the significance they bring to the fundamental understanding of soliton physics, soliton molecules also present the possibility of transferring optical data surpassing the limitation of binary coding [2]. Recently, soliton molecules have been became one of the most challenging study field, which have been investigated theoretically and observed experimentally in some fields [3–8]. In 2005, soliton molecules were experimentally observed in dispersion-managed optical fibers [3]. In 2017, the evolution of femtosecond soliton molecules resolved in the cavity of a few-cycle mode-locked laser by means of an emerging timestretch technique [7]. In 2018, Liu et al. have experimentally observed the real-time dynamics of the entire buildup process of stable soliton molecules for the first time [8]. Two-soliton bound states in Bose-Einstein condensates with contact atomic interactions and some dynamic phenomena of soliton molecules were reported in [9, 10]. In

2019, Lou [11] introduced a velocity resonant mechanism to form soliton molecules and asymmetric solitons for three-fifth order systems. Very recently, soliton molecules and some hybrid solutions involving Lump, breather, and positon have been investigated for some  $(1 + 1)$ -dimensional and  $(2 + 1)$ -dimensional equations by Hirota bilinear method and Darboux transformation [12–27].

As we all know, the study of symmetry is one of the most powerful methods for differential equations. The symmetry group direct method has been developed to obtain full symmetry groups for some PDEs [28–31]. Once the full symmetry group of a given system is given, the Lie point symmetry group can be derived, and sometimes, the discrete transformation group can also be expected. At the same time, the related Lie point symmetries can be recovered simply by restricting arbitrary functions or arbitrary constants in infinitesimal forms. Furthermore, one can reproduce a general group invariant solutions by the full symmetry groups and a known simple solution.

In this paper, we will investigate the following KdV-Sawada-Kotera-Ramani equation [32–34]:

$$u_t + a(3u^2 + u_{xx})_x + b(15u^3 + 15uu_{xx} + u_{xxxx})_x = 0, \quad (1)$$

which was proposed to describe the resonances of solitons in a one-dimensional space [32]. The conservation law for Eq. (1) was further given by Konno [34]. The KdV-Sawada-Kotera-Ramani equation (1) is a linear combination of the KdV equation and the Sawada-Kotera equation. When  $b=0$ , Eq. (1) is reduced to the KdV equation. When  $a=0$ , Eq. (1) is reduced to the Sawada-Kotera equation [35]. In Ref. [36], the authors investigated Lie symmetries, exact solutions, and integrability to the KdV-Sawada-Kotera-Ramani equation. To our knowledge, soliton molecules and full symmetry group of Eq. (1) have not been investigated so far.

The rest of paper is organized as follows. In Section 2, by introduce a velocity resonant condition, soliton molecules are constructed from  $N$ -solitons of the KdV-Sawada-Kotera-Ramani equation. The transmission and collision properties of soliton molecules are discussed. In Section 3, the full symmetry group of the KdV-Sawada-Kotera-Ramani equation is derived by the symmetry group direct method. From the full symmetry group, a general group invariant solution can be obtained from a known solution. In Section 4, short conclusions are given.

## 2. Soliton Molecules

The bilinear form of Eq. (1) is as follows

$$(D_x D_t + aD_x^4 + bD_x^6)f \cdot f = 0, \quad (2)$$

under the transformation  $u = 2(\ln f)_{xx}$ , where  $D$  is the Hirota's bilinear differential operator and  $f = f(x, t)$  is a real function of variables  $\{x, t\}$ . Based on the Hirota's bilinear theory, the  $N$ -soliton solutions for Eq. (1) can be obtained as

$$u = 2(\ln f)_{xx}, \quad (3)$$

$$f = \sum_{\rho=0,1} \exp\left(\sum_{1 \leq j < i \leq N} \rho_i \rho_j A_{ij} + \sum_{j=1}^N \rho_j \eta_j\right),$$

with

$$\eta_i = k_i x + w_i t + \phi_i,$$

$$e^{A_{ij}} = -\frac{(k_i - k_j)(w_i - w_j) + a(k_i - k_j)^4 + b(k_i - k_j)^6}{(k_i + k_j)(w_i + w_j) + a(k_i + k_j)^4 + b(k_i + k_j)^6},$$

$$w_i = -(ak_i^3 + bk_i^5), 1 \leq j < i \leq N, \quad (4)$$

where  $\{k_i, w_i, \phi_i\} (i = 1, 2, \dots, N)$  are arbitrary constants,  $\sum_{1 \leq j < i \leq N}$  is the summation of all possible pairs taken from  $N$  elements with the condition  $1 \leq j < i \leq N$ , and  $\sum_{\rho=0,1}$  indicates a summation over all possible combinations of  $\rho_i, \rho_j = 0, 1 (i, j = 1, 2, \dots, N)$ .

To find nonsingular analytical resonant excitation from Eq. (3), we apply the velocity resonance conditions ( $k_i \neq \pm k_j, w_i \neq \pm w_j$ ),

$$\frac{k_i}{k_j} = \frac{w_i}{w_j} = \frac{bk_i^5 + ak_i^3}{bk_j^5 + ak_j^3}. \quad (5)$$

Then, we can get the following expression

$$k_i = \frac{\sqrt{-b(bk_j^2 + a)}}{b}, \text{ or,} \quad (6)$$

$$k_i = -\frac{\sqrt{-b(bk_j^2 + a)}}{b}.$$

It can be known under the resonance condition (5) that two solitons,  $i$ th soliton and  $j$ th soliton in Eq. (3), exhibit one soliton molecule structure. To see this fact, we take  $N=2$  as a simple example. When  $N=2$ , the solution (3) can be simplified to

$$u = 2[\ln(1 + e^{\eta_1} + e^{\eta_2} + A_{12}e^{\eta_1 + \eta_2})]_{xx},$$

$$\eta_i = k_i x - (bk_i^5 + ak_i^3)t + \phi_i,$$

$$A_{12} = -\frac{(k_1 - k_2)(-bk_1^5 + bk_2^5 - ak_1^3 + ak_2^3) + a(k_1 - k_2)^4 + b(k_1 - k_2)^6}{(k_1 + k_2)(-bk_1^5 - bk_2^5 - ak_1^3 - ak_2^3) + a(k_1 + k_2)^4 + b(k_1 + k_2)^6}. \quad (7)$$

Figure 1 displays the molecule structure expressed by Eq. (7) with the parameter selections

$$\begin{aligned} a &= 6, \\ b &= -1, \\ k_1 &= 1, \\ k_2 &= -\sqrt{5}, \\ \phi_1 &= 0, \\ \phi_2 &= 4. \end{aligned} \quad (8)$$

From Figure 1, one can see that two solitons in the molecule are different because  $k_1 \neq k_2$  though the velocities of them are same.

If changing values  $\phi_1$  and  $\phi_2$ , the distance between two solitons of the molecule will change, respectively. When the distance of two solitons is close enough to have an interaction with each other, the soliton molecule will become an asymmetric soliton. Figure 2 is the plots of the asymmetric soliton with parameters (8) except for  $\{\phi_1 = -8, \phi_2 = 10\}$ . From Figure 2, one can see the two-soliton molecule keeps its asymmetric shape and velocity during the evolution.

Two-soliton molecules can be generated from four solitons;  $k_1, w_1$  and  $k_2, w_2$  satisfy Eq. (6);  $k_3, w_3$  and  $k_4, w_4$  satisfy Eq. (6) at the same time. Figure 3 displays the elastic

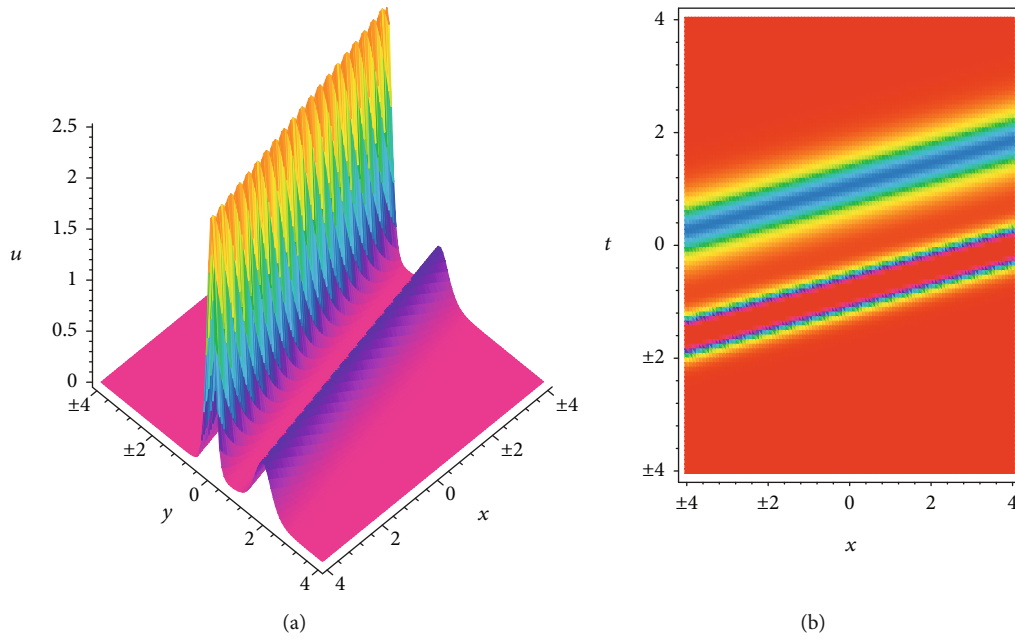


FIGURE 1: Soliton molecule structure for Eq. (1) with the parameter selections (8): (a) three-dimensional plot; (b) density plot.

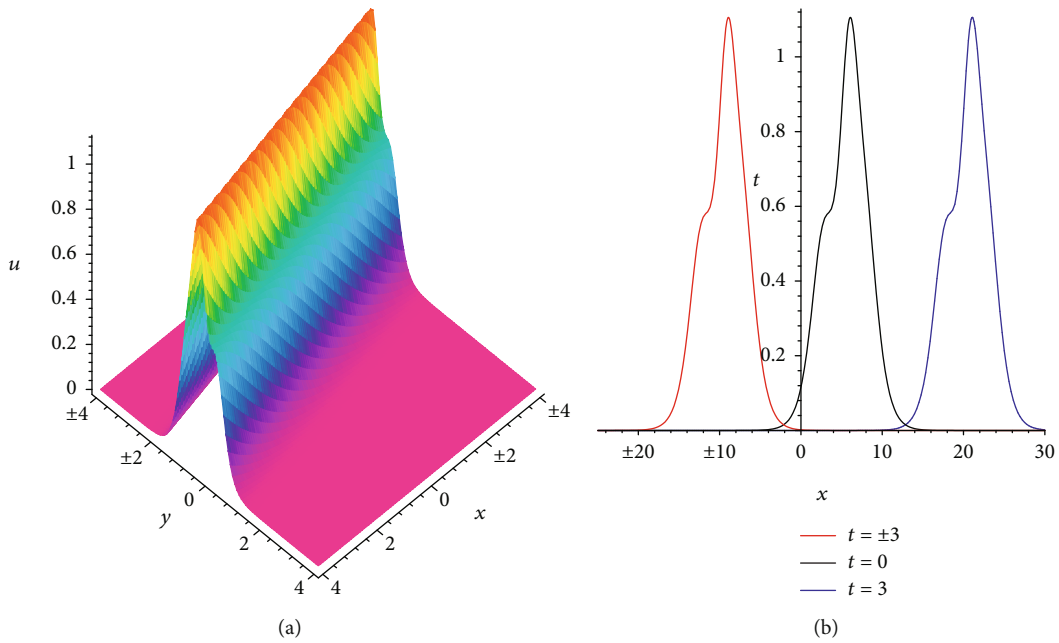


FIGURE 2: Asymmetric soliton for Eq. (1) with the parameter selections (8) except for  $\{\phi_1 = -8, \phi_2 = 10\}$ : (a) three-dimensional plot; (b) two-dimensional plot at  $t = -3, 0, 3$ .

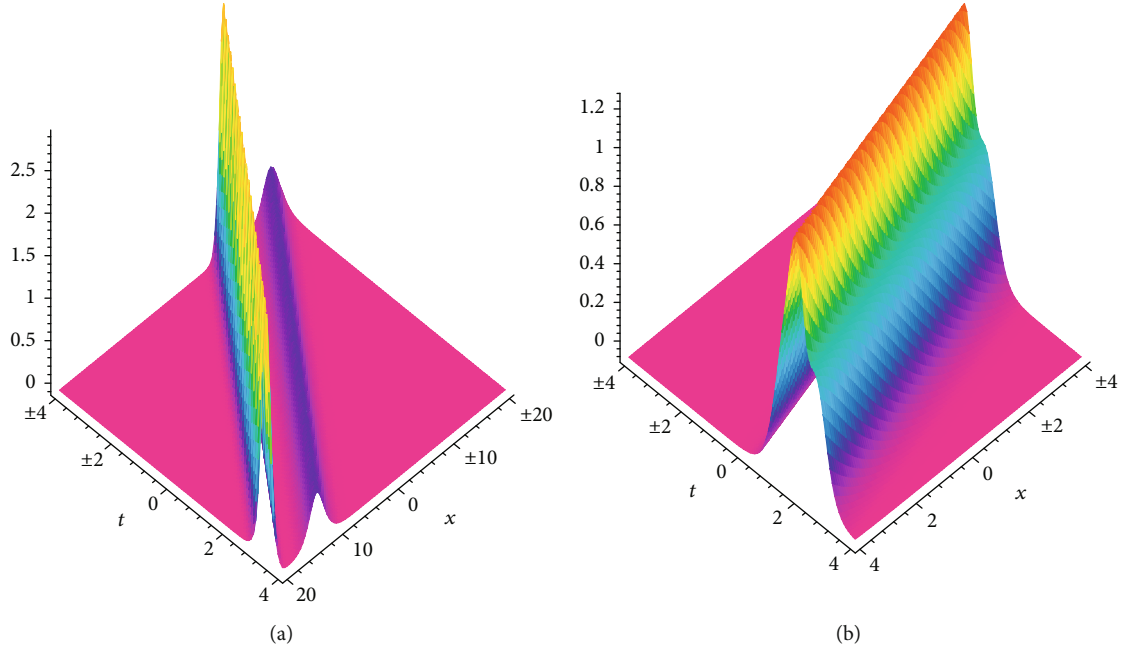


FIGURE 3: Three-dimensional plots of solution (17) with parameter selections (8) and (18): (a) soliton molecule structure; (b) asymmetric soliton structure.

interaction property for the solution (3) with  $N = 4$  and with parameter selections

$$\begin{aligned}
 a &= 6, \\
 b &= -1, \\
 k_1 &= 1, \\
 k_2 &= -\sqrt{5}, \\
 \phi_1 &= 0, \\
 \phi_2 &= 6, \\
 k_3 &= \frac{3}{4}, \\
 k_4 &= -\frac{\sqrt{87}}{4}, \\
 \phi_3 &= 4, \\
 \phi_4 &= 8 \left( k_3 = \frac{1}{2}, k_4 = -\frac{\sqrt{23}}{2}, \phi_3 = -2, \phi_4 = 10 \right).
 \end{aligned} \tag{9}$$

From Figure 4, one can see that the height of wave peaks and velocities are not changed except for phase after collision.

### 3. Finite Symmetry Groups

According to the symmetry group direct method, we set the solutions of Eq. (1) as follows:

$$u = \alpha + \beta U(\xi, \tau), \tag{10}$$

where  $\alpha = \alpha(x, t)$ ,  $\beta = \beta(x, t)$ ,  $\xi = \xi(x, t)$ ,  $\tau = \tau(x, t)$ , and  $U = U(\xi, \tau)$  satisfy the same equations in Eq. (1)

$$U_\tau + a(3U^2 + U_{\xi\xi})_\xi + b(15U^3 + 15UU_{\xi\xi} + U_{\xi\xi\xi\xi})_\xi = 0, \tag{11}$$

Substituting Eq. (11) into Eq. (1) and eliminating all terms including  $U_{\xi\xi\xi\xi}$  by Eq. (11), we obtain one polynomial differential equations with respect to  $U$  and their derivatives. Then, collecting the coefficients of  $U$  and their derivatives, we obtain a set of overdetermined partial differential equations with respect to differential functions:  $\{\alpha, \beta, \xi, \tau\}$ . From the overdetermined PDEs, it is easy to find that

$$\begin{aligned}
 \tau &= \tau(t), \\
 \beta &= \beta(t), \\
 \xi_{xx} &= 0, \\
 \alpha_x &= 0.
 \end{aligned} \tag{12}$$

Now, the substitution of Eq. (12) into the overdetermined PDEs leads to

$$\begin{aligned}
 \beta(\tau_t - \xi_x^5) &= 0, \\
 \beta(45\xi_x \alpha^2 b + 6\xi_x \alpha a + \xi_t) &= 0, \\
 6\beta\xi_x(-a\xi_x^4 + 15b\alpha\beta + a\beta) &= 0, \\
 -\beta\xi_x^3(a\xi_x^2 - 15b\alpha - a) &= 0, \\
 15\beta\xi_x^3 b(\beta - \xi_x^2) &= 0, \\
 45b\beta\xi_x(\beta - \xi_x^2)(\beta + \xi_x^2) &= 0, \\
 \alpha_t &= \beta_t = 0.
 \end{aligned} \tag{13}$$

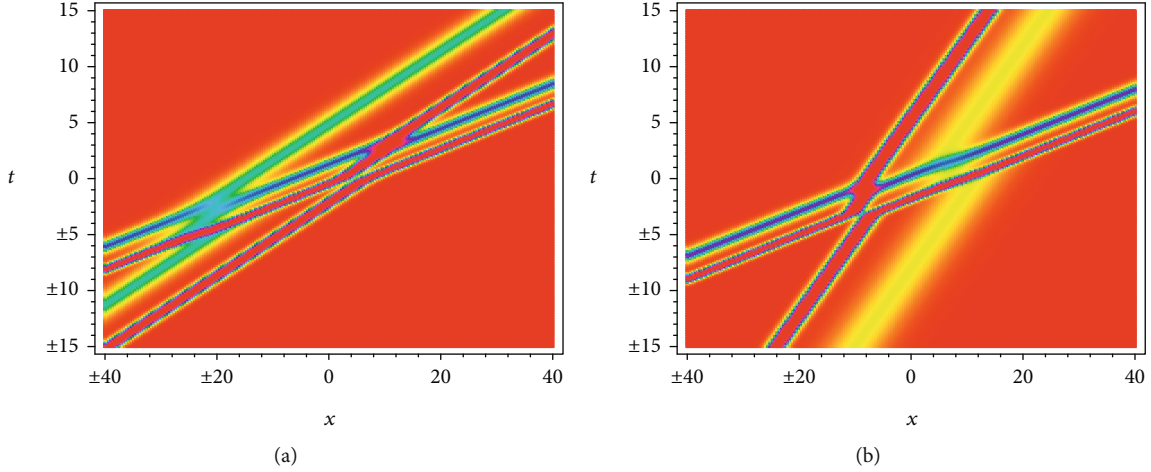


FIGURE 4: Two-soliton molecule structure for the solution (3) of Eq. (8) with the parameter selections (9).

With the help of symbolic computation after performing some calculations, the general solutions of Eqs. (12) and (13) are as follows

$$\begin{aligned} \alpha &= \frac{a(\sigma^2 c^2 - 1)}{15b}, \\ \beta &= c^2 \sigma^2, \\ \xi &= \sigma c x - \frac{a^2(\sigma^4 c^5 - c)}{5b} t + \xi_0, \tau = c^5 t + \tau_0, \end{aligned} \quad (14)$$

where  $b \neq 0$ ,  $c$ ,  $\xi_0$ , and  $\tau_0$  are arbitrary constants while the constant  $\sigma$  possess discrete values determined by

$$\sigma^5 = 1 \Rightarrow \begin{cases} \sigma_1 = 1, \\ \sigma_2 = \frac{(\sqrt{5} - 1) + i\sqrt{10 + 2\sqrt{5}}}{4}, \\ \sigma_3 = \frac{(\sqrt{5} - 1) - i\sqrt{10 + 2\sqrt{5}}}{4}, \\ \sigma_4 = \frac{(\sqrt{5} - 1) + i\sqrt{10 - 2\sqrt{5}}}{4}, \\ \sigma_5 = \frac{(\sqrt{5} - 1) - i\sqrt{10 + 2\sqrt{5}}}{4}, \quad (i = \sqrt{-1}). \end{cases} \quad (15)$$

From the above results, one can get the following symmetry group theorem for Eq. (1).

**Theorem 1.** If  $U = U(x, t)$  is a solution of KdV-Sawada-Kotera-Ramani equation (1), then so is

$$u = \frac{a(\sigma^2 c^2 - 1)}{15b} + \sigma^2 c^2 U(\xi, \tau), \quad (16)$$

with (14) and (15).

From the symmetry group Theorem 1, we know that for the real KSKR equation, the Lie point symmetry group which corresponds to  $\sigma = 1$ . For the complex KSKR equation, the symmetry group is divided into five sectors which correspond to five values of  $\sigma$  in Eq. (15). At the same time, we can derive the classical Lie symmetry from Theorem 1 by taking arbitrary constants  $\{c, \xi_0, \tau_0\}$  as some special infinitesimal parameter forms.

Furthermore, one can obtain a general group invariant solutions of KSKR equation by Theorem 1 and a known simple solution. For example, from Eq. (7) and Theorem 1, we can derive a general two solitons for KSKR equation as follows

$$\begin{aligned} u &= \frac{a(\sigma^2 c^2 - 1)}{15b} + 2\sigma^2 c^2 [\ln(1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2})]_{\xi\xi}, \\ \eta_i &= k_i \xi - (b k_i^5 + a k_i^3) \tau + \phi_i, \end{aligned} \quad (17)$$

with (14) and (15).

It is necessary to point out that when two-soliton solution (17) exhibits one soliton molecule structure, the velocity resonance condition is the same as (5). Figure 3 displays the molecule structure and asymmetric soliton structure expressed by (17) with the parameter selections (8) and

$$\begin{aligned} \sigma &= 1, \\ c &= 1.1, \\ \xi_0 &= 2(\xi_0 = 0), \\ \tau_0 &= 1(\tau_0 = -2). \end{aligned} \quad (18)$$

## 4. Conclusion

In this paper, we investigated the KdV-Sawada-Kotera-Ramani (KSKR) equation, which is used to simulate the resonances of solitons in one-dimensional space. On the basis of general  $N$ -soliton express and velocity resonance mechanism, we obtained one soliton molecule and multiple soliton

molecule structure for the KSKR equation. Asymmetric soliton can be formed by adjusting the distance between two solitons to small enough. The interactions among these soliton molecules for the KSKR equation are elastic. Then, we derived the full symmetry group for the KSKR equation by the symmetry group direct method. From the full symmetry group, a general group invariant solution can be obtained from a known solution. The results of the paper can be expected to provide some useful information for the dynamic behaviors of KSKR equations. It is necessary to note that on the basis of  $N$ -soliton solutions by Hirota bilinear method, we only obtain a soliton molecule including two solitons for the KSKR equation. It is usually not easy to derive a soliton molecule containing multiple solitons. In order to obtain a soliton molecule containing multiple solitons, one can investigate  $N$ -solitons from Darboux transform to find a soliton molecule with multiple solitons [14]. The method to construct soliton molecules and the symmetry group method can be applied to investigate other nonlinear models.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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